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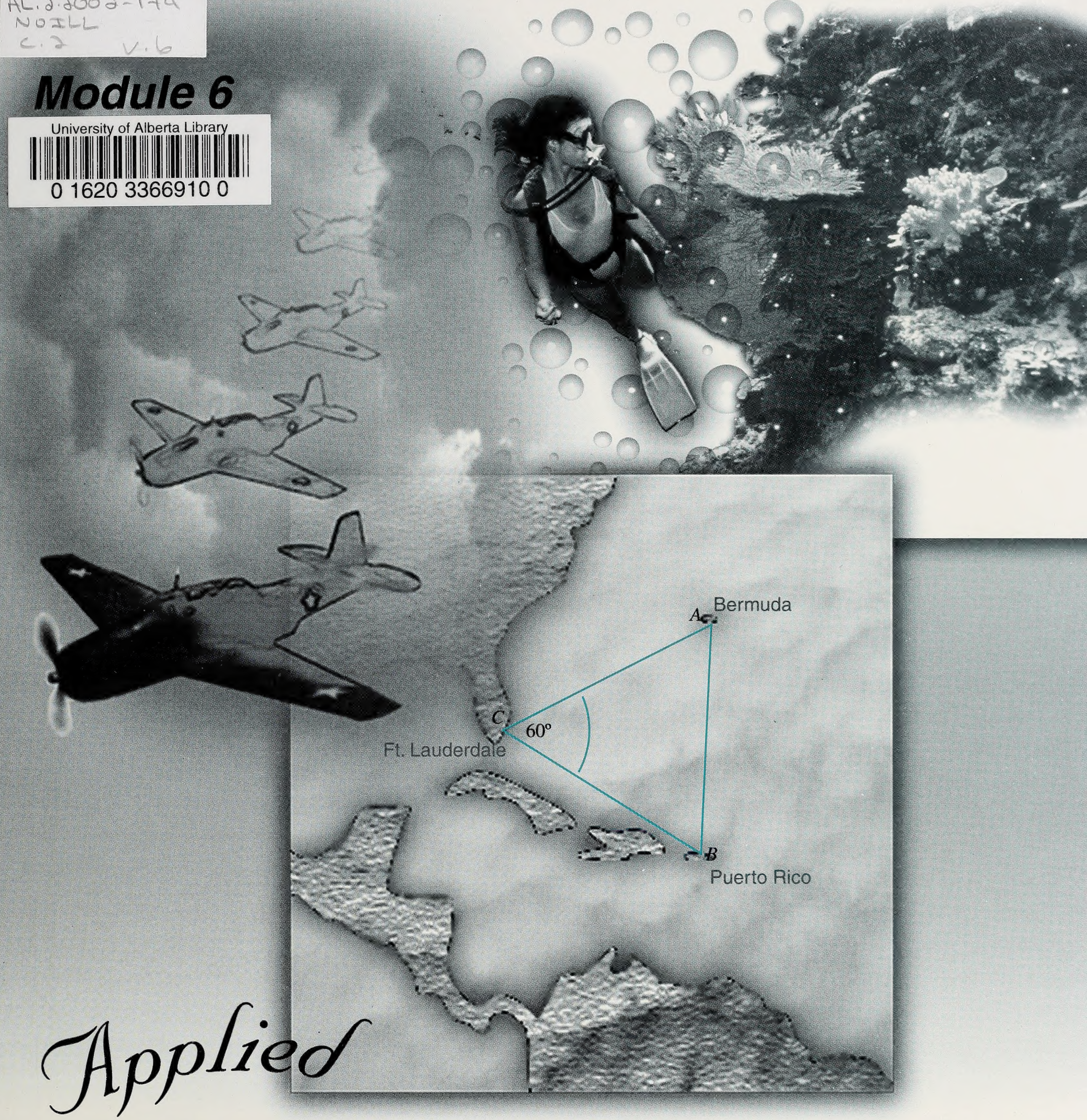

Applied Mathematics 10

LINEAR FUNCTIONS

Learning
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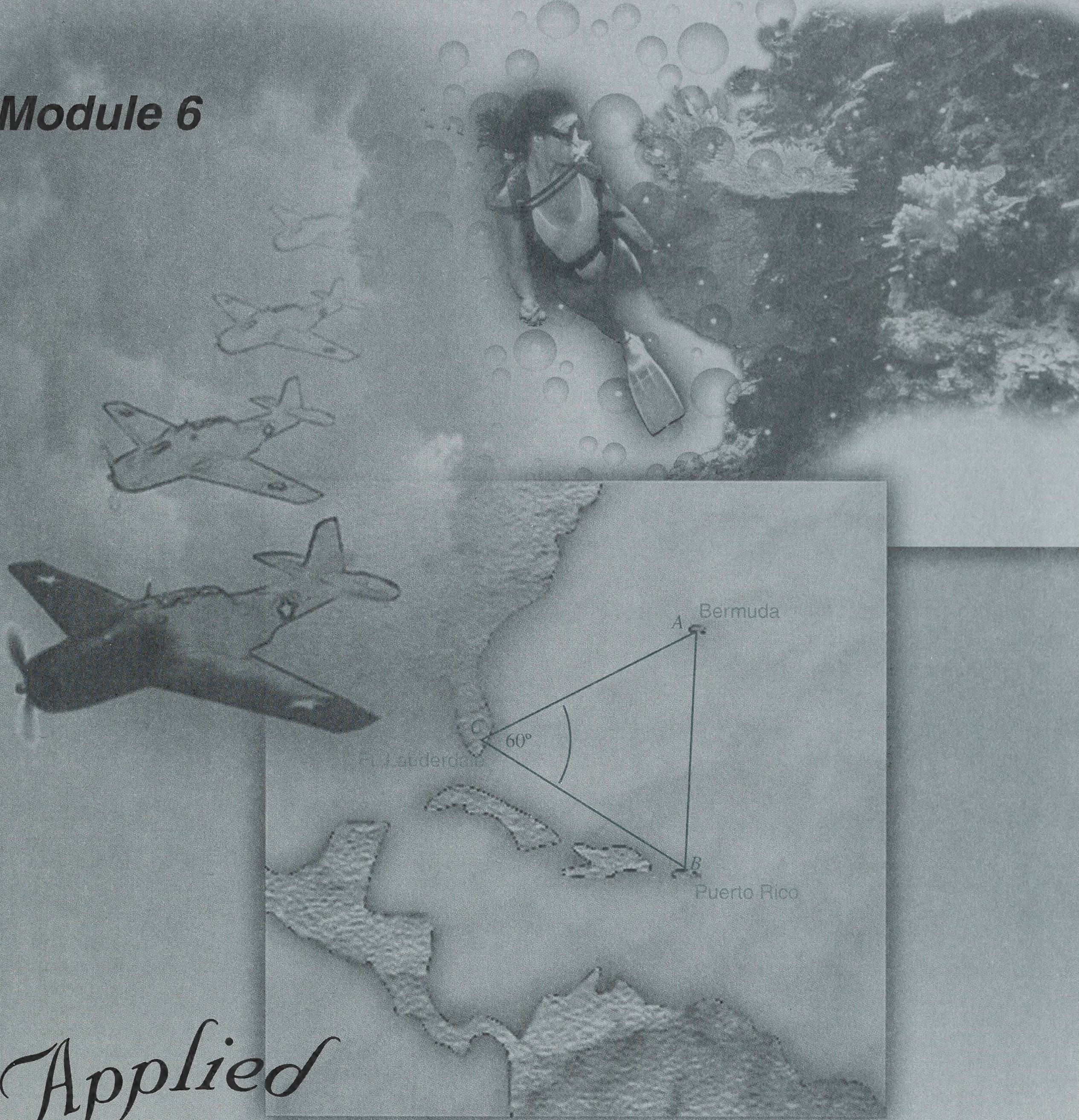


Applied Mathematics 10

LINEAR FUNCTIONS

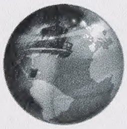
Learning Technologies Branch

Alberta LEARNING

Applied Mathematics 10
Student Module Booklet
Module 6
Linear Functions
Learning Technologies Branch
ISBN 0-7741-1797-4

Cover photo: COREL CORPORATION

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	



The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://www.learning.gov.ab.ca/ltb

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Welcome

Applied Mathematics 10

Module 1
Measurement

Module 2
Number Patterns in Tables

Module 3
Relations and Functions

Module 4
Sampling

Module 5
Line Segments

Module 6
Linear Functions

Module 7
Trigonometry

Welcome to Module 6. We hope you'll enjoy your study of Linear Functions.

Applied Mathematics 10 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

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Submit the Module Assignment.

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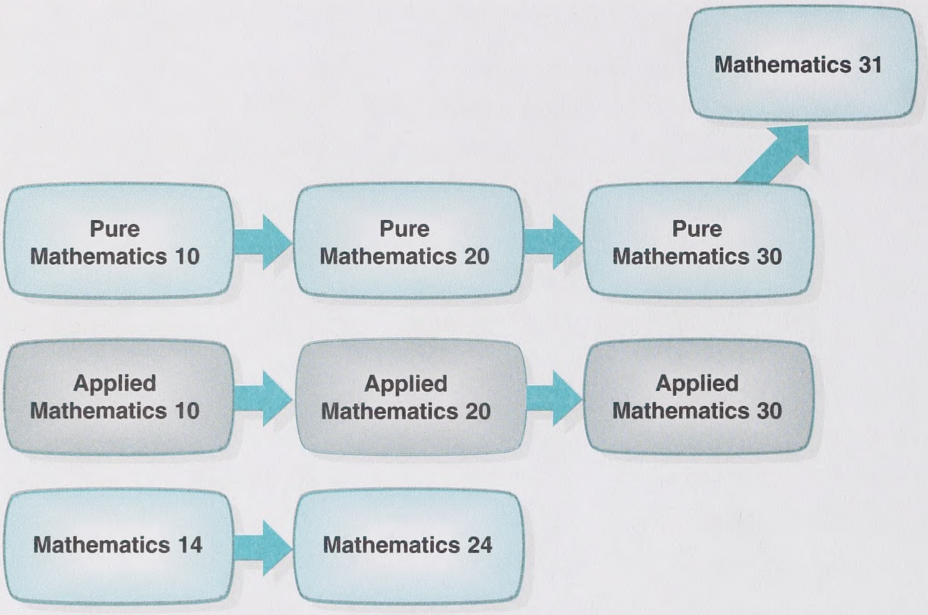
Introduction to Applied Mathematics 10

Applied Mathematics 10 is the first course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different post-secondary and career plans.

You may find it helpful to read mathematics updates on Alberta Learning's website:

http://www.learning.gov.ab.ca

You may also wish to visit the website for the Learning Technologies Branch:

http://www.learning.gov.ab.ca/ltb

Before enrolling in Applied Mathematics 10, it is recommended that you talk with a school counsellor about your career plans.

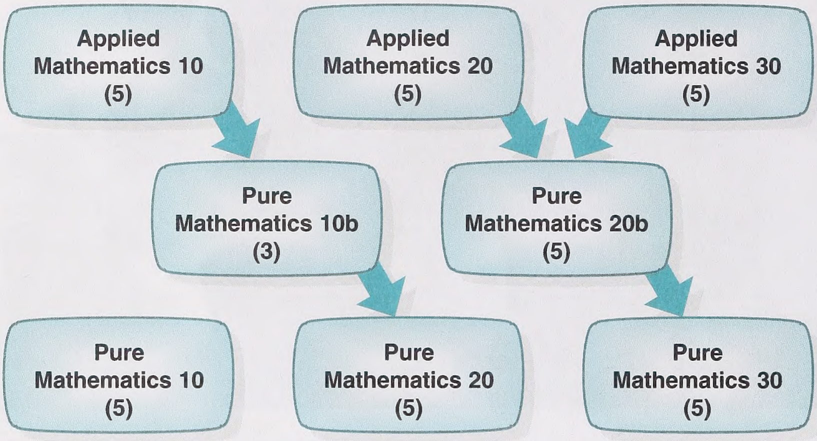
TRANSFERRING FROM THE APPLIED MATHEMATICS PROGRAM

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are specific to either the applied or pure mathematics courses.

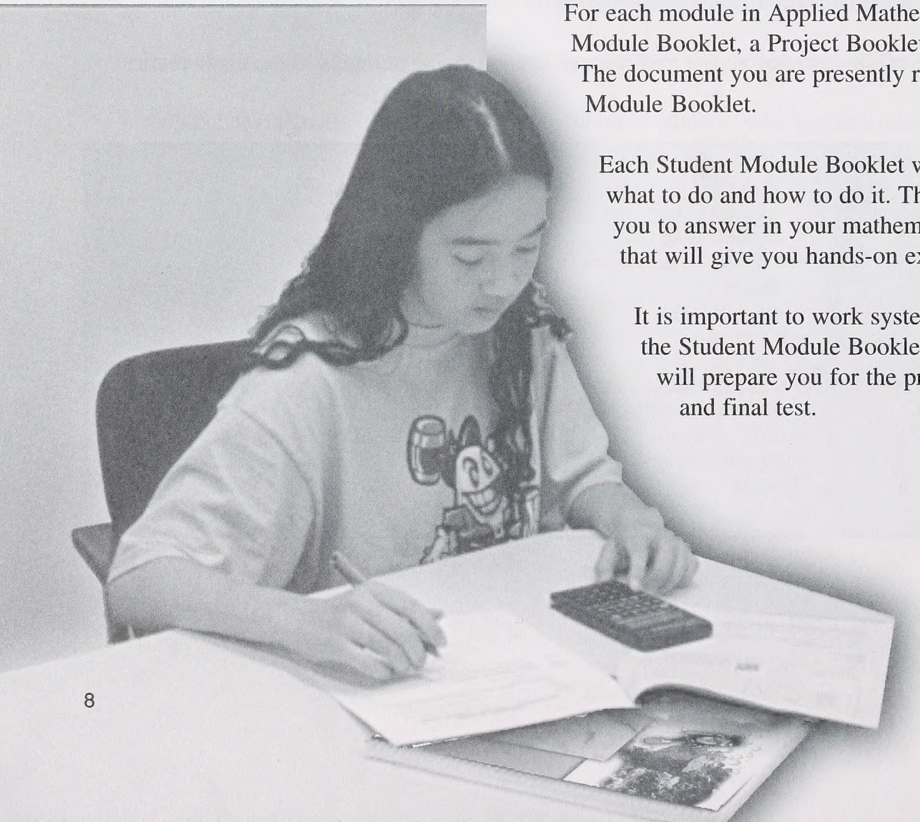
The following table shows some of the common topics and some of the independent topics in the mathematics program.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may take the 3-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may take the 5-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.



STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 10



For each module in Applied Mathematics 10, there is a Student Module Booklet, a Project Booklet, and an Assignment Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the projects, assignments, and final test.

Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also, store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words, and recording useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

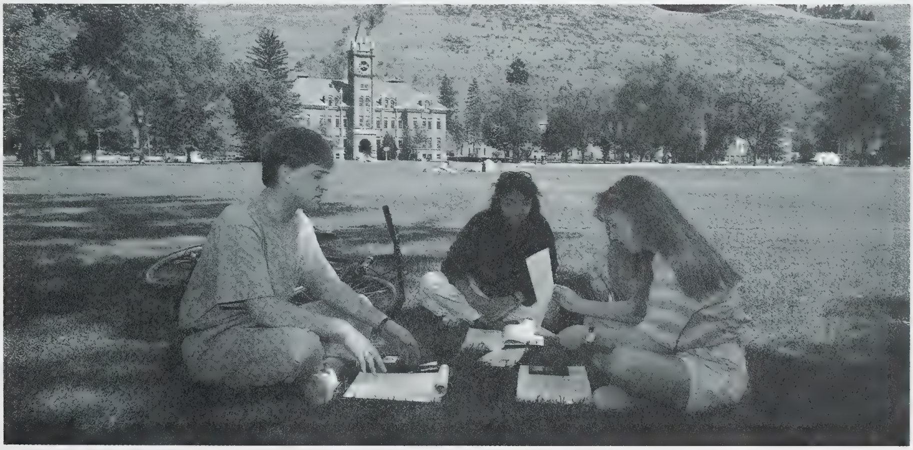
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience) or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 10 to be your study partner. You will find that having a friend with whom you can discuss mathematical ideas will make your studying more enjoyable.



Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 10, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 10 Source Book,* Western Canadian Edition, published by Addison Wesley Longman Ltd. (1999)

- a binder, lined loose-leaf paper, graph paper, dividers, a pencil, and an eraser

- metric and imperial measuring devices, such as a ruler, a tape measure, a yardstick, a vernier caliper, and a micrometer

- a mathematical instrument set (compass, protractor, and triangles)

- a computer installed with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and *Microsoft® Excel*. The examples in this course show *Microsoft® Excel.*

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Brands	Texas Instruments	Sharp	Casio
Models	TI-82 TI-83 TI-83 Plus TI-86 TI-89 TI-92 TI-92 Plus	EL-9600c EL-9600* EL-9300* EL-9200*	CFX-9850Ga-Plus CFX-9850G* CFX-9800G* FX-9700 series*

*no longer commercially available, but may be available on loan from your school division

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you view the video *The TI-83 Graphing Calculator Video Tutor* to discover some of the calculator's features.

Many of the resources you will need for this course may be purchased from the Learning Resources Distributing Centre (LRDC). Following is the LRDC website:

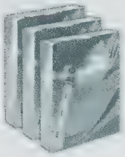
http://www.lrdc.edc.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

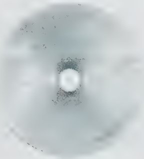
Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the explanations given to discover what the various icons prompt you to do.

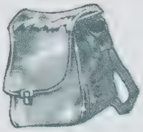
- Refer to the textbook.



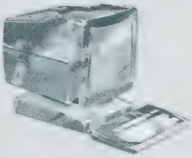
- Use the companion CD for Applied Mathematics 10.



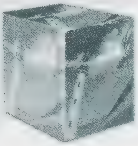
- Use mathematical instruments, measuring devices, and other materials.



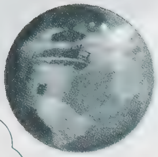
- Work with a computer.



- Complete the module project or assignment.

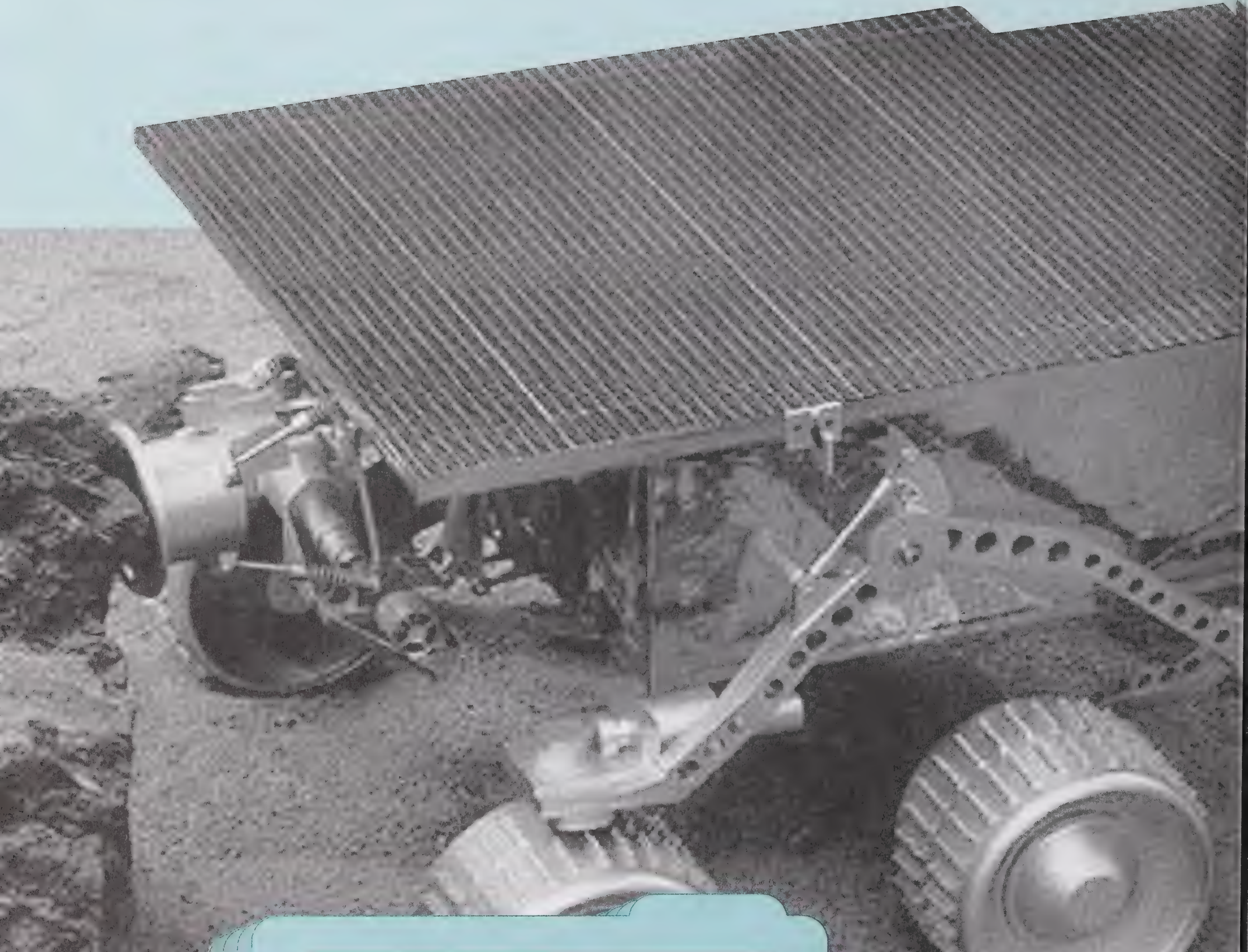


- Explore the Internet.

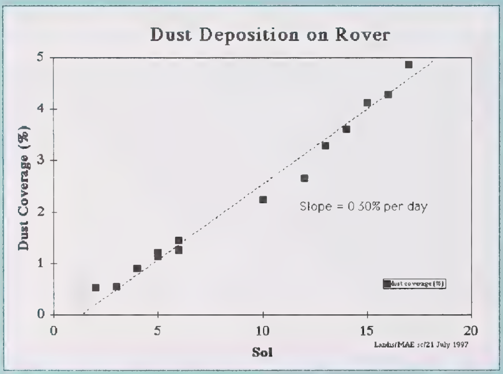


Remember: Any Internet website address given in this module is subject to change.

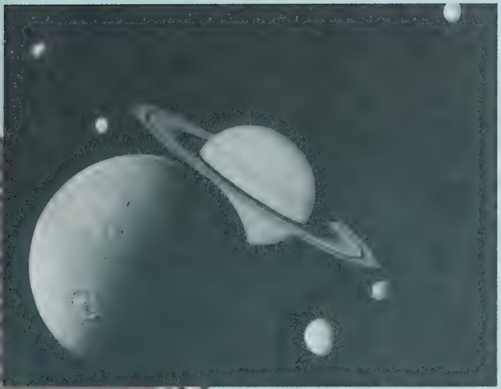
MODULE OVERVIEW



NASA



NASA



Dust seems to settle everywhere. If you don't clean your room thoroughly, dust becomes visible on the top of your dresser and other surfaces. Computer-moniter screens and TV screens seem to accumulate dust very quickly.

It is difficult to escape dust. There is dust even beyond Earth's atmosphere. On the Mars Pathfinder mission in 1997, the amount of dust settling on a glass surface of the Sojourner Rover (shown in the photo) was measured indirectly by the change in transparency of a glass surface. Scientists had estimated that an additional 0.22% of the total glass area would be covered by dust every Mars day (sol). The actual data in the graph indicate a rate of 0.30% per sol—very close to the prediction. Because there was little variation in the rate of deposit when the Rover was moving or still, it was concluded that the dust was in the atmosphere.

JPL NASA

The data of the rate of dust deposition on the Sojourner Rover approximates a linear function. In this module, you will derive the characteristics of a linear function from its equation. You will see the connection between linear functions and direct variations and arithmetic sequences. You will determine the line of best fit for a set of data. You will use a correlation coefficient to indicate the degree to which a set of data fits a linear function.

EVALUATION

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation. The mark distribution is as follows:

Module Project	**30 marks**
Module Assignment	**70 marks**
TOTAL	**100 marks**

Remember that Activities 1 to 3 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

MODULE PROJECT
CONTRASTING TWO SPECIES

Beginning the Project

You may have heard the comment that people tend to look like their pets. While certain similarities may exist, an animal's form differs from a human form. The functions of various body parts may differ too.

The form of a body is its shape or structure. The function of a body or its parts is its purpose or what it is to be used for. For example, think of a frog. A frog's hind legs can be described as big and muscular—that is the form. An important function of the legs is to allow the frog to move by hopping. Think of a porcupine. Having quills is an aspect of its form. The function of the quills is to protect the porcupine.

Your module project for Module 6: Linear Functions is called Contrasting Two Species. Your project will focus on the form and function of two species: humans and gorillas.

In this project, you will investigate the characteristics of a gorilla to determine aspects of its form. Then, you will infer connections between the animal's form and its function. In order to make the connection between form and function, you will need to know how and where gorillas live. You will also collect some data on humans in order to have a standard for comparison.

In preparing for this project, turn to the Project Booklet and preview what data and other information you are expected to submit.

Turn to page 254 of the textbook and read "Form and Function." Start your research as indicated, using the listed questions to get you started. Keep your research notes in the project section of your mathematics binder.

You may wish to begin your Internet research by visiting the Addison Wesley Longman Ltd. site, described on page 255 of the textbook. You may also find many other Internet sites on lowland gorillas. You can learn about various aspects of gorillas at these sites.

Your project will be marked on accuracy of information, mathematical representation of data, clarity of language, and validity of inferences. In this project, focus on the measurement data associated with gorillas and humans. In order to describe and analyse the data, you will need a good understanding of scatterplots and linear functions.

As you work through the activities in this module continue your research for this project. You will be given more direction on how to complete this project later in this module. In the meantime, you may wish to discuss this project with others; but, remember, the work you submit must be your own.

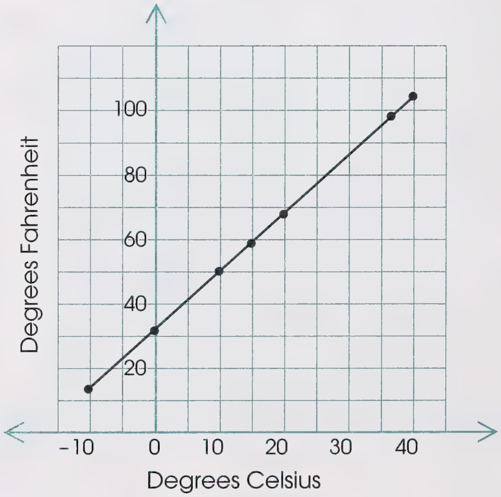
ACTIVITY 1

Linear Functions

A temperature reading can quickly and conveniently determine whether or not you have a fever. Normal body temperature (taken by mouth) is considered to be 37°C or 98.6°F. A higher temperature reading on either scale indicates a fever.

The following chart shows some readings in Celsius and their equivalent readings in the Fahrenheit scale. The relation between the values can also be shown as a **scatterplot**.

°C	°F
−10	14
0	32
10	50
15	59
20	68
37	98.6
40	104



The points in the scatterplot lie on a line. Any additional data points based on this relation would also lie on the line. The relation between degrees Fahrenheit and degrees Celsius is a linear function.

A **linear function** is a function whose equation can be written in the form $y = mx + b$ and whose graph consists of points on a straight, non-vertical line (these points may or may not be joined).

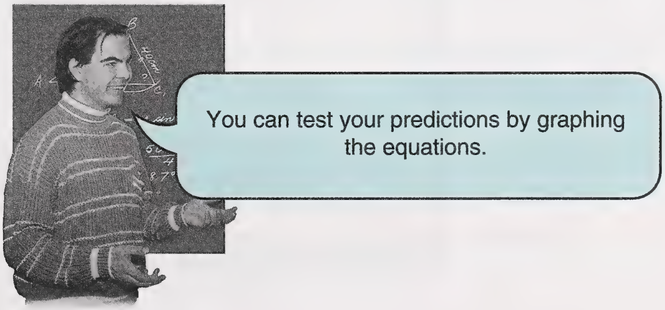
Turn to page 256 of the textbook and complete questions 1 to 7 of "Investigation 1: Relationship between Arm Span and Height." Store your answers in the project section of your mathematics binder. **Note:** If you do not have a large enough group with which to work, supplement your data with information from the following table.

Name of Student	Arm Span (cm)	Height (cm)
Aladdin	170	158
Arif	180	165
Bill	140	160
Meghan	175	182
Pat	195	180
Tyrel	96	100
Udham	95	92
Wayne	130	110

GRAPHING FUNCTIONS

As shown in the comparison of Celsius and Fahrenheit temperatures, you can represent a linear function by a table of values or a scatterplot. However, a more concise way to describe a linear function is by using a suitable equation.

Turn to page 257 of the textbook and examine the equations in exercise 1 of "Investigation 2: Linear Functions." Predict which of the equations represent linear functions. Keep a record of your predictions.


You can test your predictions by graphing the equations.

Work through the following example.

Example

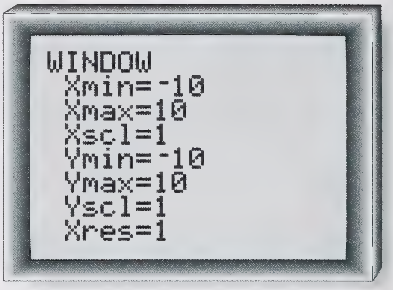
Use a graphing calculator to determine if the equation $y=2x+3$ represents a linear function.

Solution

Step 1: Set the WINDOW to the standard settings.

Press ZOOM 6 WINDOW.

Note: If a graph from a previous entry appears when you press ZOOM 6, press WINDOW **after** the graph has been completed.



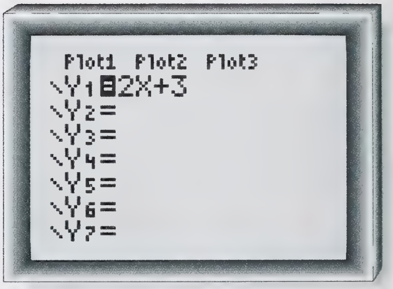
```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

Step 2: Enter the equation.

Press Y=.

Use the arrow keys and CLEAR to delete any previous entries.

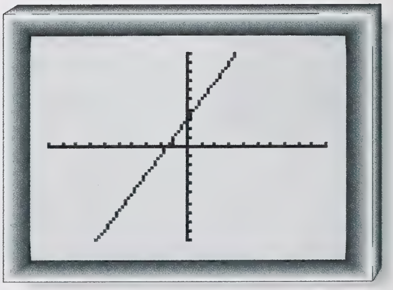
Press 2 X,T,Θ,n + 3.



```
 Plot1 Plot2 Plot3
\Y1=2X+3
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Step 3: Display the graph of $y=2x+3$.

Press GRAPH.



Since the graph of $y=2x+3$ is a straight, non-vertical line, the equation represents a linear function.

Work through another example. Predict whether or not the equation $y=\frac{1}{x}, x\neq 0$ represents a linear function.

Example

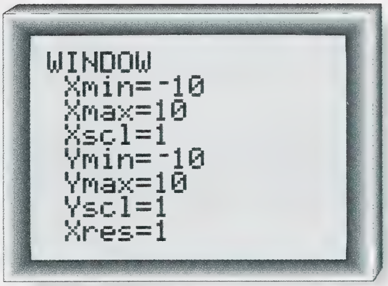
Use a graphing calculator to determine if the equation $y=\frac{1}{x}, x\neq 0$ represents a linear function.

Solution

Step 1: Set the WINDOW to the standard settings.

Press ZOOM 6 WINDOW.

Recall: If necessary, press WINDOW **after** a graph from a previous entry appears.



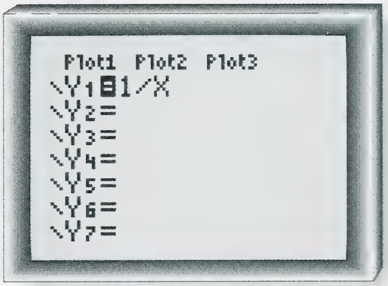
```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

Step 2: Enter the equation.

Press Y=.

Use the arrow keys and CLEAR to delete any previous entries.

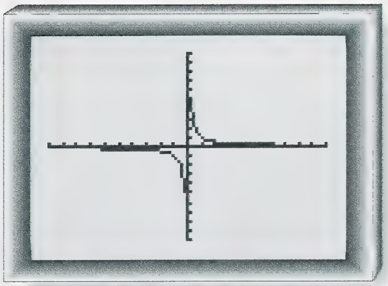
Press 1 ÷ X,T,Θ,*n*.



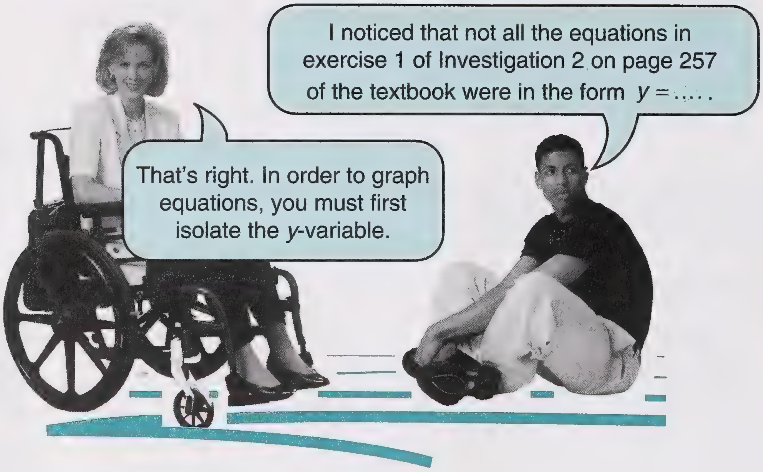
```
 Plot1  Plot2  Plot3
\Y1=1/X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Step 3: Display the graph of $y=\frac{1}{x}, x\neq 0$.

Press GRAPH.



Since the graph of $y=\frac{1}{x}, x\neq 0$ does not form a straight line, the equation does not represent a linear function.


I noticed that not all the equations in exercise 1 of Investigation 2 on page 257 of the textbook were in the form y = ...
That's right. In order to graph equations, you must first isolate the y-variable.

Work through the following example to see how the *y*-variable is isolated.

Example

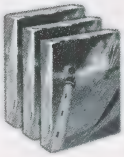
Rearrange the following equations in order to isolate the *y*-variable to the left of the equal sign.

a. $5x+y-4=0$ **b.** $3x+4y=2$

Solution

a.
$$\begin{aligned} 5x+y-4&=0 \\ 5x+y&=4 && \longleftarrow \text{Add 4 to each side.} \\ y&=-5x+4 && \longleftarrow \text{Subtract } 5x \text{ from each side.} \end{aligned}$$

b.
$$\begin{aligned} 3x+4y&=2 \\ 4y&=-3x+2 && \longleftarrow \text{Subtract } 3x \text{ from both sides.} \\ y&=-\frac{3}{4}x+\frac{1}{2} && \longleftarrow \text{Divide both sides by 4.} \end{aligned}$$



1. Turn to page 257 of the textbook and answer exercises 2 and 4 of "Practise Your Prior Skills."

Compare your responses with the suggested answers in the Appendix, Activity 1, page 79.

With the y-variable isolated, an equation is ready for graphing and investigation.

2. **a.** Turn to page 257 of the textbook and graph the equations in exercise 1 of "Investigation 2: Linear Functions." (Omit the equations $y = 2x + 3$ and $y = \frac{1}{x}, x \neq 0$. These equations were graphed in the preceding examples.) If necessary, first isolate the y-variable.

State whether each graph represents a linear function. Compare your responses with the predictions you made earlier about which equations represent linear functions.

b. What similarities did you find in the equations that represented straight lines?

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 79–80.

Read the definition of a linear function on page 258 of the textbook. When you isolated the y-variable in the linear equations in question 2, you put the equation in the slope y-intercept form, $y = mx + b$. The following example illustrates how to read the values of m and b from the $y = mx + b$ form.

$$\begin{aligned} y - 2x &= -6 \\ y &= 2x - 6 \\ y &= 2x + (-6) \end{aligned}$$

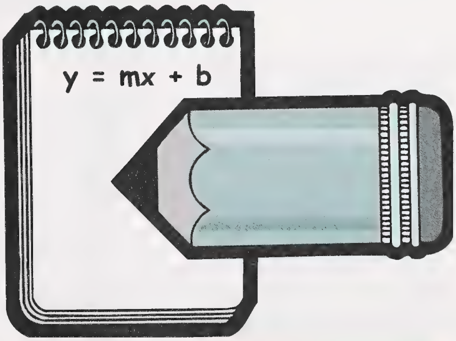
$$m = 2;\ b = -6$$

3. Write the following equations in $y = mx + b$ form and state the values of m (the slope) and b (the y-intercept) in each equation.

a. $y = -4x + 3$

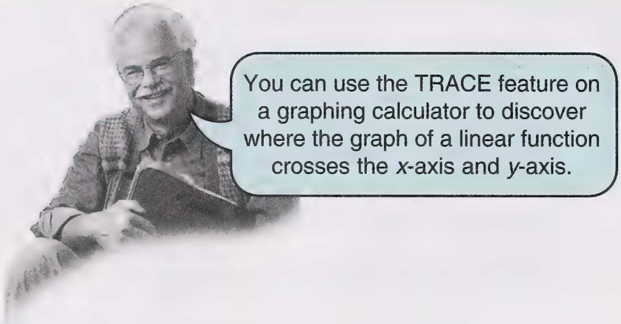
b. $y + 5 = 3x$

c. $y = 0.25x - 1$



Compare your responses with the suggested answers in the Appendix, Activity 1, page 81.

FINDING THE INTERCEPTS


You can use the TRACE feature on a graphing calculator to discover where the graph of a linear function crosses the *x*-axis and *y*-axis.

Work through the following example.

Example

Graph the equation $10x + 5y = 32$ using your graphing calculator. Then, use the TRACE feature to find

- the *y*-coordinate of the point where the graph crosses the *y*-axis
- the *x*-coordinate of the point where the graph crosses the *x*-axis

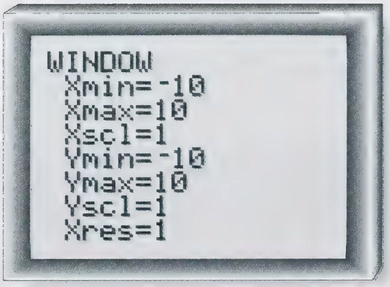
Solution

Step 1: Rearrange the equation so the *y*-variable is isolated.

$$\begin{aligned} 10x + 5y &= 32 \\ 5y &= -10x + 32 && \leftarrow \text{Subtract } 10x \text{ from both sides.} \\ y &= -2x + 6.4 && \leftarrow \text{Divide both sides by 5.} \end{aligned}$$

Step 2: On your graphing calculator, choose the standard WINDOW settings.

Press ZOOM 6 WINDOW.



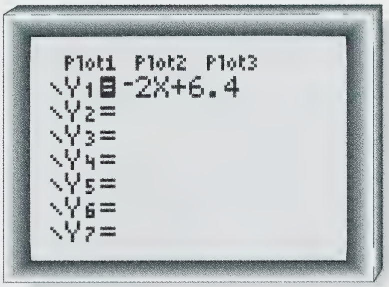
```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

Step 3: Enter the equation to be graphed and graph the function.

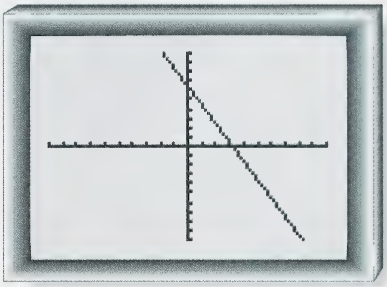
Press Y= . If necessary, clear any entries. Then, enter the equation $y = -2x + 6.4$ and graph it.

Press (–) 2 X,T,Θ,*n* + 6 • 4 GRAPH .

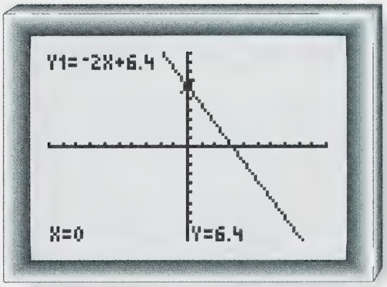
This is the grey negative key, not the blue subtraction key.



```
 Plot1  Plot2  Plot3
\Y1=-2X+6.4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



Step 4: Press TRACE .



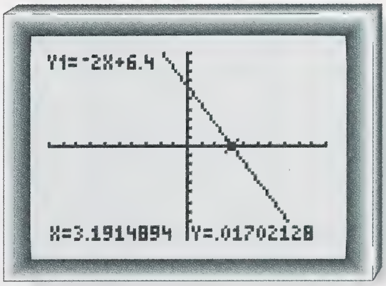
```
Y1=-2X+6.4
X=0           Y=6.4
```

If necessary, use the arrow keys to move the cursor along the graph onto the *y*-axis. (You will see a flashing cursor on the graph and the *x*-value will be 0.)

The *y*-coordinate of the point where the graph crosses the *y*-axis is 6.4.

Step 5: With the arrow keys, move the cursor along the graph onto the *x*-axis—if possible (the *y*-value will be 0).

The *y*-coordinate of the traced point closest to the *x*-axis is 0.017 021 28.



```
Y1=-2X+6.4
X=3.1914894   Y=.01702128
```

Did you find that it is not possible to move the cursor precisely onto the *x*-axis? On the *x*-axis, the *y*-value is 0. So, the *x*-value showing on the screen is not the actual *x*-coordinate of the crossing point.

Note: In this example, the TRACE feature did not produce the correct *x*-value. How can the TRACE feature be used more effectively?

The graphing calculator used in this example has 94 pixels (dots) across the calculator screen. Using the standard WINDOW setting (Xmin set to -10 and Xmax set to 10), the spread is 20 and each pixel represents $\frac{20}{94}$, which is approximately 0.212 766. Because of the long decimal, the standard WINDOW setting may not be convenient for tracing.

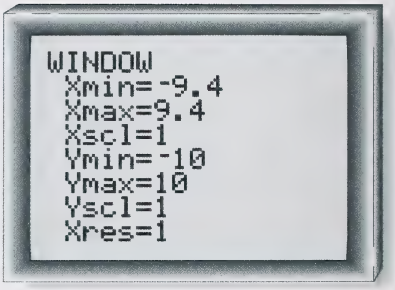
A "friendly" WINDOW setting—that is, one using a multiple of 94—is better for tracing. One example of a "friendly" WINDOW setting is Xmin set to -9.4 and Xmax at 9.4. The spread is now 18.8, and each pixel represents $\frac{18.8}{94}$, which is 0.2.

Recall the current example. By re-doing the problem with a "friendly" WINDOW setting, you may be able to trace directly onto the *x*-intercept.

Step 1: Press WINDOW. Enter the "friendly" settings.

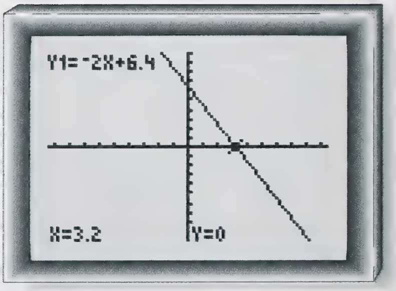
Set Xmin to -9.4 and Xmax to 9.4.

Press GRAPH.



```
WINDOW
 Xmin=-9.4
 Xmax=9.4
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

Step 2: Press TRACE and use the arrow keys to move the cursor along the graph onto the *x*-axis (the *y*-value will be 0). Using the "friendly" settings allows you to move the cursor precisely onto the *x*-axis.



```
Y1=-2X+6.4
X=3.2        Y=0
```

The *x*-coordinate of the point where the graph crosses the *x*-axis is 3.2.

Turn to page 257 of the textbook and read the information in the middle of the page about a different “friendly” window.

Remember, even the “friendliest” WINDOW setting may not allow you to find the *x*- or *y*-intercepts in all cases. You may have to make further WINDOW adjustments in order to use the TRACE feature successfully.

4. Turn to page 258 of the textbook and answer exercises 1 to 4 of “Investigation 3: Find the Intercepts.” **Note:** Use a “friendly” WINDOW setting on your graphing calculator.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 81–84.

In your investigation, you solved equations with a graphing calculator to find where the graphs of linear functions cross the *x*-axis and *y*-axis.

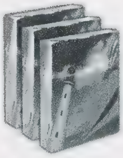
- The coordinates of all points on the *y*-axis have ordered pairs that look like (0, ▨). The *y*-coordinate of the point where a graph intersects the *y*-axis is called the ***y*-intercept**.
- The coordinates of all points on the *x*-axis have ordered pairs that look like (▨, 0). The *x*-coordinate of the point where a graph intersects the *x*-axis is called the ***x*-intercept**.

Using the TRACE feature of your graphing calculator is one way to find the *x*- and *y*-intercepts.

Changing the Values of *m* and *b* in a Linear Equation

You have found that an equation of the form $y = mx + b$ stands for a linear function. You can use various values for *m* and *b* in the $y = mx + b$ form of the equation for a linear function.

Consider the following. What happens to the graph of $y = mx + 2$ as *m* decreases? That is, what happens when the coefficient of *x* decreases and the rest of the equation remains the same? You may want to make a prediction. In the next investigation, you will see what happens when *m* takes on the values $2, 1, 0.5, 0, -1,$ and -3, while *b* remains the same.



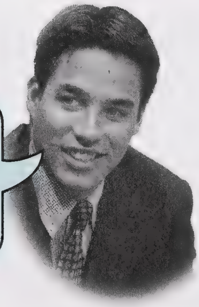
5. Turn to page 260 of the textbook and answer exercises 1 to 10 of "Investigation 1: Changing *m*."

Note: There are seven functions in the investigation, not six; enter the functions as Y1 to Y7. Use the standard WINDOW setting on your graphing calculator. Enter the functions in order of decreasing values of *m*.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 85–86.

You discovered in working through Investigation 1 that making the value of *m* decrease while *b* remains the same makes the graph rotate clockwise around one point—the point where each graph crossed the *y*-axis.

The variable *m* in the general linear equation $y = mx + b$ corresponds to the slope.



Recall the definition of slope of a line segment from Module 5. The **slope of a line** is equal to the slope of any line segment on the line.

What happens to the graph when *b* is changed in the linear equation $y = 2x + b$? Do you have any prediction? In the next investigation, *b* is made to decrease through the values $5, 3, 1, 0, -2,$ and -4, and the value of *m* is kept constant.

6. Turn to page 261 of the textbook and answer exercises 1 to 8 of "Investigation 2: Changing b." **Note:** Use the standard WINDOW setting on your graphing calculator. Enter the equations in order of decreasing values of b.

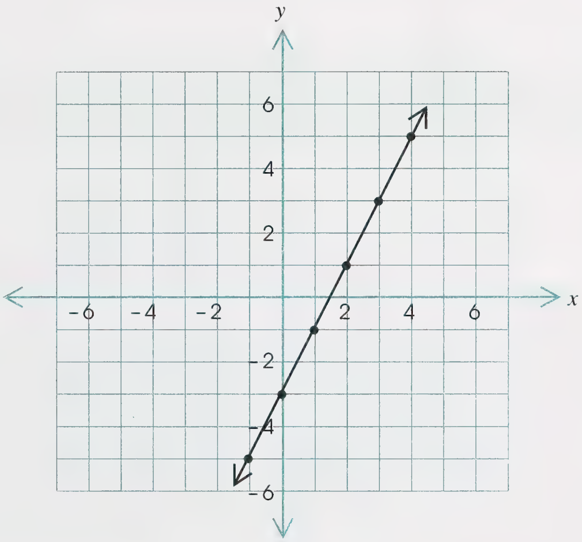
Compare your responses with the suggested answers in the Appendix, Activity 1, page 87.

Your investigation showed that changing the value of b affects where the graph crosses the y-axis; the value of b in $y = mx + b$ is the y-intercept.

Turn to page 261 of the textbook and read the text (in the shaded box) in the middle of the page. Then, answer the following questions.

7. Express the linear equation $3x - 4y + 20 = 0$ in slope y-intercept form.

8. Use the following graph to answer parts a., b., and c.

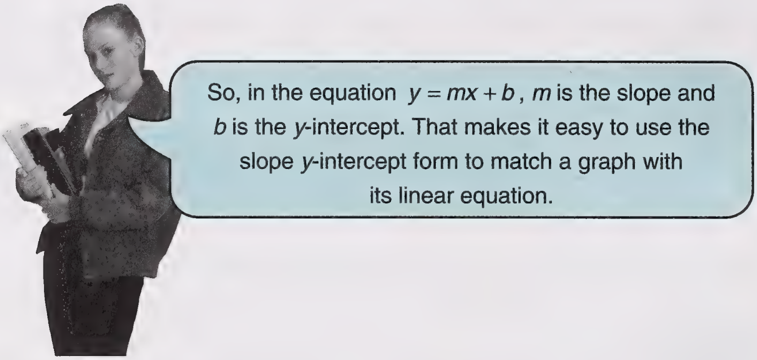


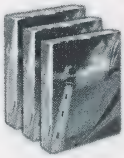
a. Determine the slope of the graph. Show your work.

b. What is the y-intercept of the graph?

c. Write the equation for the graph in the slope y-intercept form.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 88.

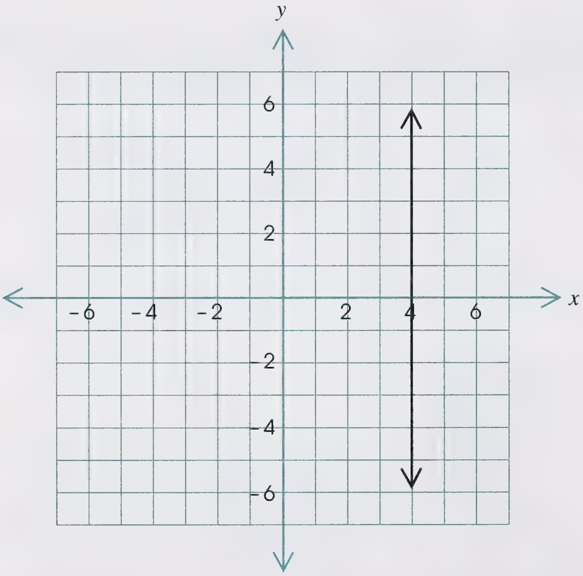

So, in the equation $y = mx + b$, m is the slope and b is the y-intercept. That makes it easy to use the slope y-intercept form to match a graph with its linear equation.



Turn to page 262 of the textbook and work through "Example: Determine the slope and intercepts of the function defined by a linear equation."

9. Turn to page 264 of the textbook and answer exercises 1.b., 1.c., 3.b., 3.e., and 3.f. of "Exercises: Checking Your Skills." **Note:** Use your graphing calculator to graph the equations in exercise 1.

10. Use the given graph to answer parts a., b., and c.

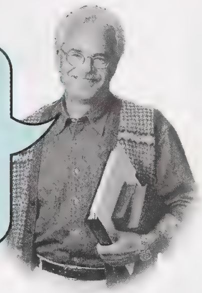


a. List the ordered pairs for any three points on the straight-line graph.
b. Give an equation for the graph.
c. Is the relation represented by the graph a function? Explain.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 61, pages 89–90.

Vertical and Horizontal Lines

You've seen that a vertical line cannot be represented in the slope *y*-intercept form. You will discover more about vertical and horizontal lines in the next investigation.



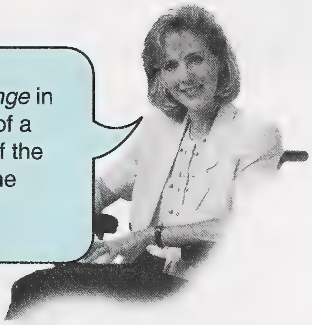
11. Turn to pages 261 and 262 of the textbook and answer exercises 1 to 8 of "Investigation 3: Horizontal and Vertical Lines." **Note:** Use the standard WINDOW settings on your graphing calculator.

12. Turn to pages 263 and 264 of the textbook and answer exercises 3 to 6 of "Discussing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 91–93.

A horizontal line can be represented by an equation of the form $y = b$; a vertical line can be represented by an equation of the form $x = a$. Both a and b are constants. A vertical line **cannot** be represented in the slope y-intercept form.

You worked with the terms *domain* and *range* in Module 3. You can think of the domain of a function as all the *x*-values of the points of the function's graph and the range as all the *y*-values of these points.



13. Turn to page 264 of the textbook and answer exercises 2, 4, and 5 of "Exercises: Checking Your Skills."

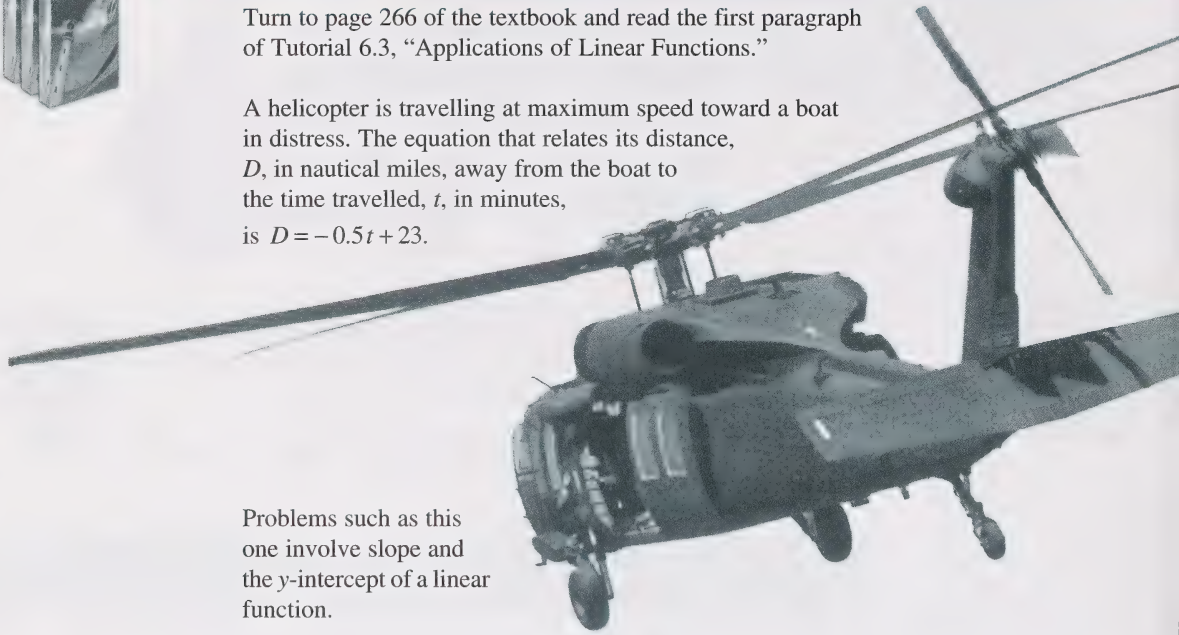
Compare your responses with the suggested answers in the Appendix, Activity 1, pages 93–96.

You should now be able to determine the intercepts, slope, domain, and range for a given linear function. This will help you in solving problems.

APPLICATIONS OF LINEAR FUNCTIONS

Turn to page 266 of the textbook and read the first paragraph of Tutorial 6.3, "Applications of Linear Functions."

A helicopter is travelling at maximum speed toward a boat in distress. The equation that relates its distance, D, in nautical miles, away from the boat to the time travelled, t, in minutes, is $D = -0.5t + 23$.



Problems such as this one involve slope and the y-intercept of a linear function.

The slope and the y-intercept of a linear function have a special significance when the function is used in problem solving.

Turn to pages 266 and 267 of the textbook and work through "Example 1: Interpreting a linear function."

Note how the variables of the linear function in the example are renamed. This readies the equation for graphing and presents the equation in a familiar form.

Independent variable: n → x

Dependent variable: C → y

Equation: $C = 25n + 100$ → $y = 25x + 100$

Vertical intercept: C-intercept → y-intercept

14. Based on the preceding information, what are two labels that can be used for the horizontal intercept of the linear relation in Example 1 (the banquet example)?

15. Turn to page 314 of the textbook and answer exercise 9 of Part B of "What Should I Be Able To Do?"

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 96–98.

Are there any circumstances that call for a negative *m*-value (for slope) in a linear equation?

Sure. Just think about a football game and consider that movement on the field may be thought of as positive or negative.

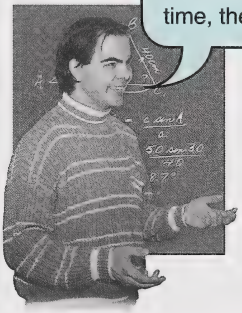
Movement towards an object or location, such as a gain in yards towards the goal posts, is considered positive. Movement away from an object or location, such as a quarterback sack, is considered negative.

In describing motion, movement towards an object or location is often considered positive, and movement away is considered negative. Movement upward is usually considered positive; downward movement is considered negative.

When using a map, the direction north is often considered positive and south is negative.

The next example uses this sign convention. The example also shows how the *x*-intercept can be interpreted.

Turn to pages 267 and 268 of the textbook and work through "Example 2: Interpretation of a linear equation."

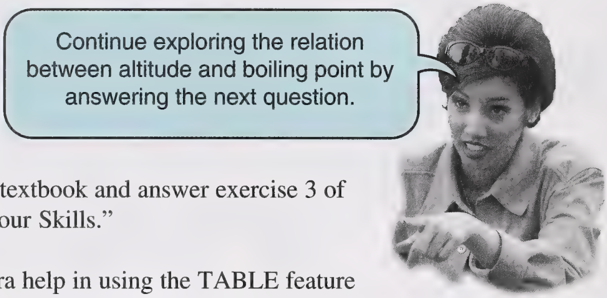


The slope of an equation used to represent relations in the real world represents a rate. When the relation involves distance and time, the slope likely corresponds to speed.

16. Turn to pages 271 and 272 of the textbook and answer exercises 1 and 2 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 98–100.

The boiling point of water varies with the height above sea level. In Victoria, British Columbia, which is at sea level, water boils at 100°C. In Calgary, Alberta, at an altitude of 1049 m, water boils at 96.4°C. At the top of Mount Logan, Yukon, the highest point in Canada at 5950 m above sea level, water boils at 79.8°C.


Continue exploring the relation between altitude and boiling point by answering the next question.

17. Turn to page 272 of the textbook and answer exercise 3 of "Exercises: Checking Your Skills."

Note: If you require extra help in using the TABLE feature on your graphing calculator, see Utility 9, "Working with Tables: Viewing a Table of Values," on page 405 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 100–102.

18. Glenice drives a tanker truck on various routes in Alberta.

She finds that when she carries crude oil, the total weight of the loaded truck is related by a linear function to the number of barrels of crude being carried.

The total weight of the truck is $m = 140v + 10\ 000$, where v is the number of barrels of oil and m is the reading, in kilograms, of the weigh scales.

a. Graph the linear function for a domain of 0 to 200 barrels. Use your graphing calculator.

b. What is the value of the vertical intercept?

c. What physical feature does the vertical intercept represent?

d. What is the slope of the relation?

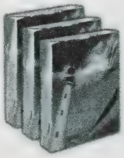
e. Interpret the slope of the relation.

f. Determine the range of the relation.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 102–103.

Hydrologists are scientists who study the behaviour of water.

Hydrologists have determined that pressure increases quickly as you go down in water. Even one metre below the surface of water, the pressure is high enough to make inhaling (through a long enough snorkel) extremely difficult.

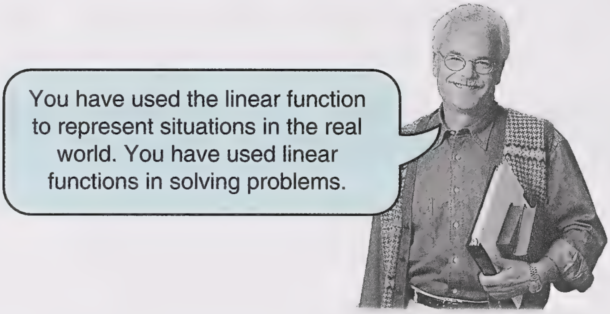


19. Turn to page 273 of the textbook and read exercise 7 of "Exercises: Checking Your Skills." Then, answer the following.

a. Why is the vertical intercept on the graph not 0?

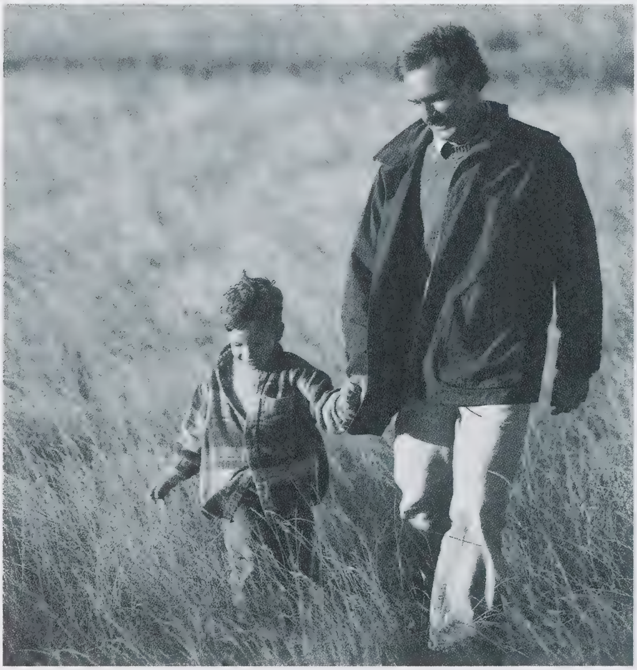
b. Explain why snorkels are not made long enough to allow dives several metres below the water's surface.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 103.


You have used the linear function to represent situations in the real world. You have used linear functions in solving problems.

DIRECT VARIATIONS AS LINEAR FUNCTIONS

You've probably heard the expression "Twice as fast, twice the distance." This expression indicates that there is a direct variation between speed and the distance travelled.



In terms of a function, a **direct variation** is a relation in which the ouput value and the input value are always in the same ratio.

20. Examine the relationship between distance walked and time walked, as shown in the following table.

Time Walked (min)	10	25	30	60
Distance Walked (km)	1	2.5	3	6

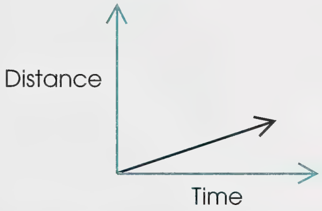
According to the table, what is the value of the ratio $\frac{\text{distance walked}}{\text{time walked}}$?

Compare your response with the suggested answer in the Appendix, Activity 1, page 104.

According to the preceding table, the ratio of the distance walked to the time walked remains constant. You can express this aspect of the relation in two ways:

- Distance walked to time walked is a direct variation.
- The distance walked varies directly with the time walked.

Suppose you let the variable d stand for the distance and the variable t for the time. Then, the direct variation for walking can be expressed by the equation $d = \frac{1}{10}t$. A sketch of the graph would look as shown. The graph is linear and includes the origin.



Features of this direct variation can be generalized to all direct variations:

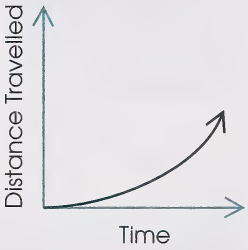
- With a direct variation in the slope y-intercept form $y = mx + b$, the value of b is 0.
- A direct variation can be expressed in an equation of the form $y = mx$, where m is a constant.
- The graph of a direct variation is linear and starts at the origin of the coordinate plane; that is, its y-intercept is 0.

Work through the following example.

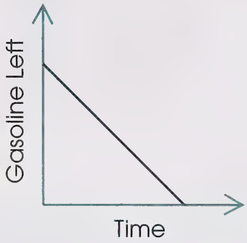
Example

Which of the following graphs represents a direct variation? Explain.

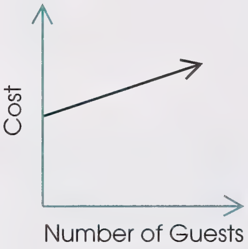
A: Dropping Marble



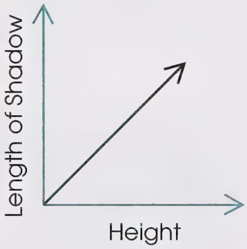
B: Car Travelling After Fill-up



C: Cost of a Banquet



D: Tree Height and Shadow Length



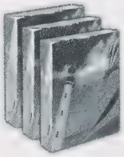
Solution

Only graph D represents a direct variation—it is linear and starts at the origin of the coordinate plane.

Graph A is not linear; it is curved upwards.

Graph B is linear, but it does not start at the origin.

Graph C is linear, but it does not start at the origin.



Turn to page 269 of the textbook and work through “Example 3: Direct variation.”

Notice that doubling the diameter of a circle doubles the circumference. Also, tripling the diameter triples the circumference. This reflects the rule for all direct variations; each of its two variables changes by the same factor.

You can use this rule to complete the next questions.

21. Josh and Pat sample some cookies. Josh eats 3 cookies, for a total of 370 cal. Pat eats 6 cookies. **Note:** Assume a direct linear relation between the number of cookies and the number of calories.

a. How many calories does Pat consume from the cookies? Explain your answer.

b. In order for the number of calories to vary directly with the number of cookies, how should the sizes of the cookies compare? Explain.

22. Turn to page 273 of the textbook and answer exercise 5 of "Exercises: Checking Your Skills."

23. Turn to page 270 of the textbook and work through "Example 4: Direct variation."

Based on this example, what would be Sam's commission on $1500 worth of computer sales? Explain your answer.

24. Turn to page 273 of the textbook and answer exercise 6 of "Exercises: Checking Your Skills." **Note:** Use your graphing calculator.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 104–108.

ARITHMETIC SEQUENCES AS LINEAR FUNCTIONS

Do you recall skip-counting by 2s, 5s, or 10s in elementary school? The sets of numbers 2, 4, 6, 8, … or 5, 10, 15, 20, … or 10, 20, 30, 40, … are sequences.

A **sequence** is a set of numbers having an order. When you are able to recognize a pattern for a sequence, then the pattern allows you to continue the sequence.

Work through the following example.

Example

Continue the following sequences. In each case, describe the pattern and continue the sequence by one more number.

a. 6, 10, 15, 21, …

b. 1, 4, 9, 16, 25, …

c. 1, 1, 2, 3, 5, …

d. 5, 8, 11, 14, …

Solution

a. The difference between consecutive numbers increases by 1.

Sequence: 6 10 15 21

$10 - 6 = 4$ $15 - 10 = 5$ $21 - 15 = 6$ $x - 21 = 7$

Next number in sequence: 28

b. Each consecutive number is a perfect square of consecutive numbers.

Sequence: 1 4 9 16 25

$1 \times 1 = 1$ $2 \times 2 = 4$ $3 \times 3 = 9$ $4 \times 4 = 16$ $5 \times 5 = 25$ $6 \times 6 = x$

Next number in sequence: 36

c. Each number, after the second, is the sum of the previous two numbers.

Sequence: 1 1 2 3 5

$1 + 1 = 2$ $1 + 2 = 3$ $2 + 3 = 5$ $3 + 5 = x$

Next number in sequence: 8

d. Each number is 3 more than the previous number.

Sequence: 5 8 11 14

$5 + 3 = 8$ $8 + 3 = 11$ $11 + 3 = 14$ $14 + 3 = x$

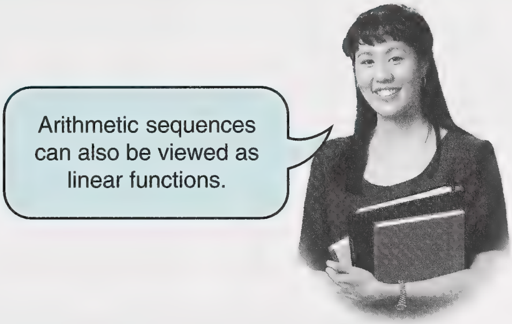
Next number in sequence: 17

Each of the numbers making up a sequence is called a **term**.

25. Turn to pages 275 and 276 of the textbook and answer exercises 1 to 7 of "Investigation 1: Number Patterns."

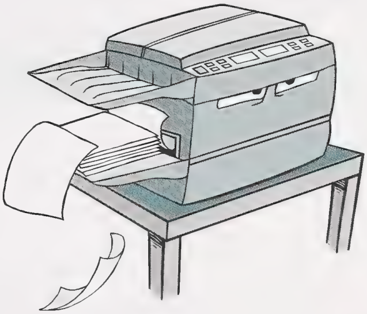
Compare your responses with the suggested answers in the Appendix, Activity 1, pages 109–110.

Using a graph to extend a sequence works most easily for sequences in which the difference between the terms is a constant value. Such a sequence is called an **arithmetic sequence**.


Arithmetic sequences can also be viewed as linear functions.

An arithmetic sequence can be represented by a linear graph and by a linear equation.

The next investigation shows how to use a linear equation for the sequence of terms representing photocopying costs. (Look to see whether the difference between the terms corresponds to the slope of the equation.)



26. Turn to page 276 of the textbook and answer exercises 1 to 8 of "Investigation 2: Photocopying Costs." You may use paper and pencil or your graphing calculator to create the table and graph for this investigation.

27. Give an example of an arithmetic sequence and its corresponding linear equation.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 111–112.

Recall that the **natural numbers** are the numbers 1, 2, 3, 4, 5, 6, … .

Turn to page 276 of the textbook and read the last two paragraphs.

28. Find the slope of the linear function that corresponds to the following arithmetic sequence:

10, 13.5, 17, 20.5, 24, …

29. The slope of the linear function corresponding to an arithmetic sequence is 12. The sequence starts with the term 6. List the next three terms. Show how you obtained your answer.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 112–113.

Turn to pages 277 and 278 of the textbook and work through "Example: Use an arithmetic sequence to solve a problem." Recognizing relations as arithmetic sequences can help in solving problems.

30. Turn to pages 278 and 279 of the textbook and answer exercises 1 and 3 of "Exercises: Checking Your Skills." **Note:** Use pencil and paper for exercise 1. Use your graphing calculator for exercise 3.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 113–115.

LOOKING BACK

In this activity, you compared linear functions to other functions, found intercepts and slopes of linear functions, interpreted horizontal and vertical lines, and used direct variations and arithmetic sequences as linear functions. You also solved a variety of problems using linear functions.

31. Turn to page 274 of the textbook and answer "Communicating the Ideas."

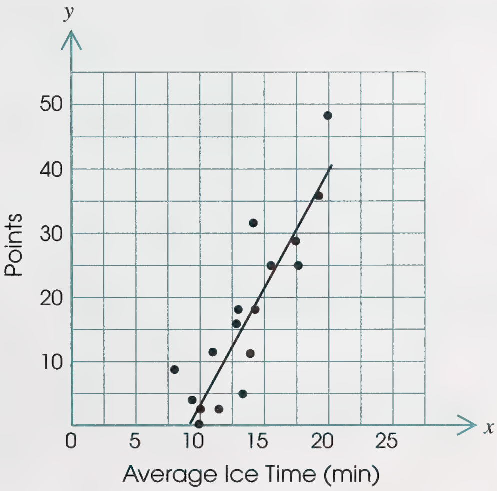
Compare your response with the suggested answer in the Appendix, Activity 1, page 115.

ACTIVITY 2

Lines of Best Fit

Do you follow hockey? Do you have a team you cheer for?

The statistics from the Edmonton Oilers, based on part of the 1999–2000 hockey season, show a trend that likely applies to forwards on your favourite team. The data suggests that forwards with more ice time make more points. It is easy to see this trend in the following scatterplot. The graph is based on the data in the chart that follows.



Player	Average Ice Time (min)	Points
Alex Selivanov	14.6	32
Bill Guerin	17.9	29
Boyd Devereaux	12.7	16
Chad Kilger	9.7	4
Daniel Cleary	11.2	3
Doug Weight	20.3	47
Ethan Moreau	14.7	18
Georges Laraque	8.3	8
Jim Dowd	12.9	12
Josef Beranek	13.8	12
Michel Picard	9.9	0
Mike Grier	16.1	24
Pat Falloon	13.3	18
Paul Comrie	10.3	3
Rem Murray	13.9	5
Ryan Smyth	18.8	36
Todd Marchant	17.6	25

The data shown in the chart is **empirical data**. Empirical data is any data that comes from observations and measurements, rather than from a rule or formula.

From the graph, you see that the empirical data lies close to, but not exactly on, a straight line. This may not be the case in all observed or measured data.

A line of best fit must be placed on the scatterplot in order to determine the associated linear function. The line of best fit shows the trend of the data. Data closest to the line of best fit can be considered to be most typical for the group.

1. Refer to the preceding graph and the table of data.

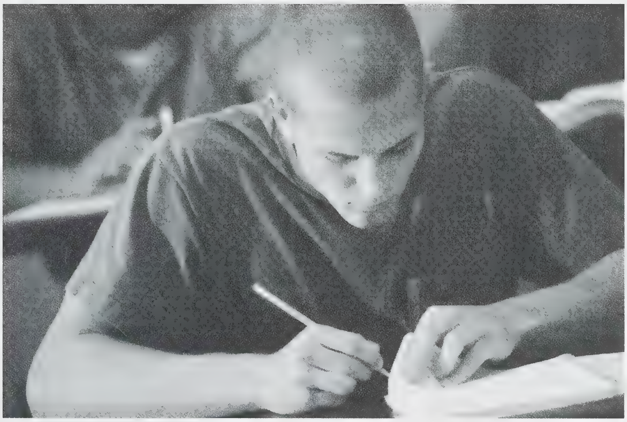
   a. Name the three Oiler forwards corresponding to data farthest above the line of best fit.

   b. Name the Oiler forward farthest below the line of best fit.

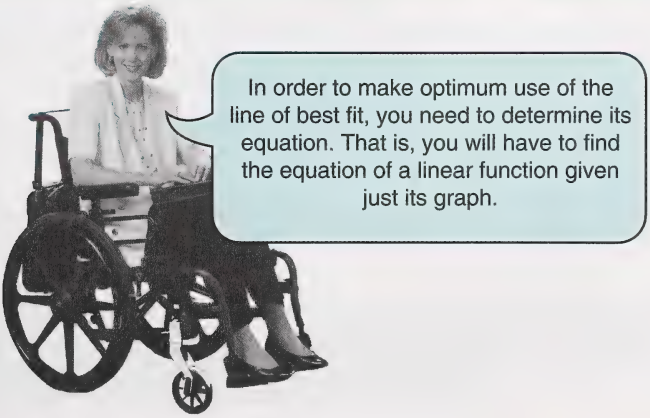
   c. Name any two Oiler forwards with "typical" performance.

Compare your responses with the suggested answers in the Appendix, Activity 2, page 116.

In this activity, you will determine the line of best fit and its equation using pencil-and-paper methods and methods involving graphing calculators or spreadsheets.



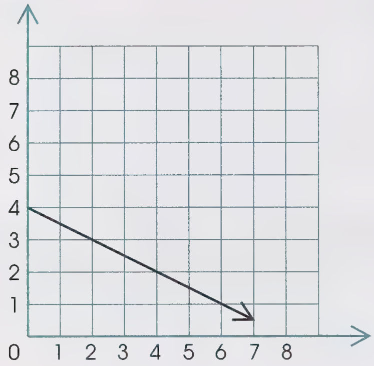
DETERMINING THE EQUATION OF A LINE



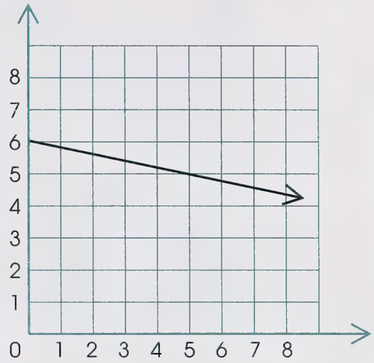
In order to make optimum use of the line of best fit, you need to determine its equation. That is, you will have to find the equation of a linear function given just its graph.

2. Select the appropriate equation from the list that follows for each of the graphs.

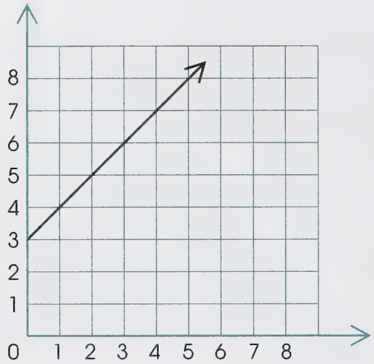
a.



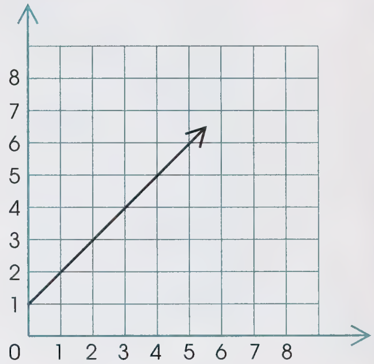
b.



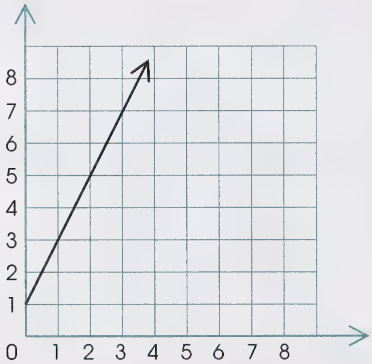
c.



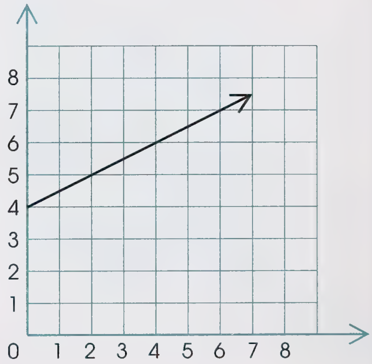
d.



e.



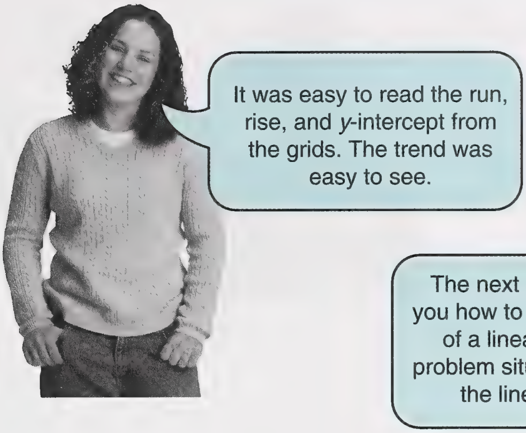
f.

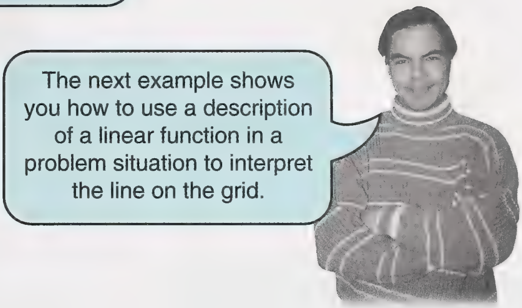


$y=-\frac{1}{2}x+4$ $\quad$ $y=-\frac{1}{5}x+6$ $\quad$ $y=\frac{1}{2}x+4$

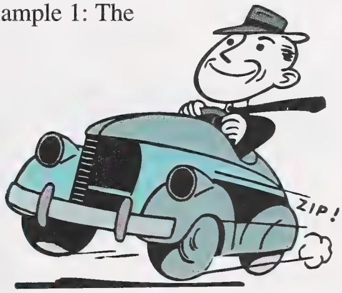
$y=x+1$ $\quad$ $y=x+3$ $\quad$ $y=2x+1$

Compare your responses with the suggested answers in the Appendix, Activity 2, page 116.


It was easy to read the run, rise, and y-intercept from the grids. The trend was easy to see.


The next example shows you how to use a description of a linear function in a problem situation to interpret the line on the grid.

Turn to page 281 of the textbook and work through "Example 1: The equation of a line given the slope and *y*-intercept."



> Notice that you obtain the values for *d* by multiplying the values you read from the graph by 100.

3. Turn to page 284 of the textbook and answer exercise 1 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 116–117.

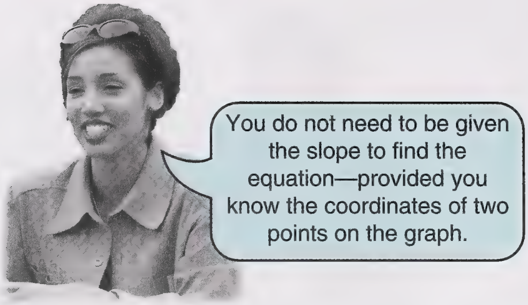
The next example shows that you do not need to be provided with the *y*-intercept to determine an equation of a linear function.

Turn to page 282 of the textbook and work through "Example 2: The equation of a linear function given the slope and one point."

4. Turn to page 284 of the textbook and answer exercises 1 and 2 of "Discussing the Ideas."

5. Turn to pages 284 and 285 of the textbook and answer exercises 2, 3.a., and 3.c. of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 117–118.


You do not need to be given the slope to find the equation—provided you know the coordinates of two points on the graph.

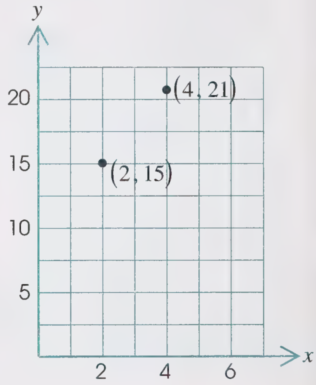
Work through the following example.

Example

Determine the equation (in x and y) for the relation containing the points $(2, 15)$ and $(4, 21)$.

Solution

The equation will be $y = mx + b$, where m and b are values to be determined.



Step 1: Determine the slope.

The slope, m, of the linear function equals the slope of the line segment joining the two points $(2, 15)$ and $(4, 21)$, since these points belong to the function.

Because you have the coordinate values of two points of the line segment, use the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{21 - 15}{4 - 2} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

Step 2: Substitute the calculated value of m in the equation $y = mx + b$.

$$y = 3x + b$$

Step 3: Solve for b. Substitute the values in point $(2, 15)$ for x and y in the equation $y = mx + b$.

$$\begin{aligned} y &= 3x + b \\ 15 &= 3(2) + b \\ 15 &= 6 + b \\ b &= 9 \end{aligned}$$

Step 4: Substitute the calculated values for m and b in $y = mx + b$.

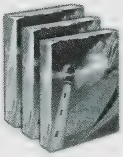
$$y = 3x + 9$$

The equation is $y = 3x + 9$.

6. Use the linear equation $y = 3x + 9$ to determine the value of y when $x = 4$.

Compare your response with the suggested answer in the Appendix, Activity 2, page 119.

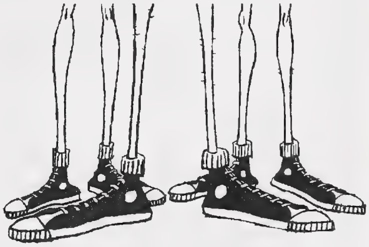
You can also determine the equation from a description that gives two sets of values for a function known to be linear. When given a description, you must interpret the information to determine the ordered pairs. You can think of each set of values as a point belonging to the function.



Turn to page 283 of the textbook and work through "Example 3: The equation of a line given two points on it."

7. Basketball player Shaquille O'Neal's shoe size is 22.

Use the equation $s = 3h - 22$ given in Example 3 to determine the length of his foot.



8. Turn to page 285 of the textbook and answer exercises 4 and 5 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 119–121.

A linear function may also involve negative values; such a function is demonstrated in the next question. Sketching graphs will help you visualize such functions.

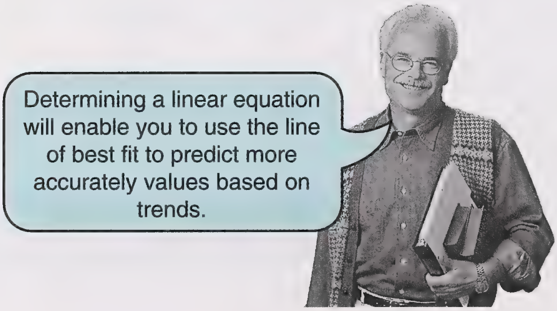
9. Turn to page 287 of the textbook and answer exercises 10.a. to 10.d. of "Exercises: Extending Your Thinking."

Hint: Since the given information starts at 6 km and -24°, you need to work from the bottom up when creating a table of values.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 122–123.

The equation of a linear function can be determined from knowing any of the following:

- the slope and *y*-intercept of the function's graph
- the slope and one point (other than the *y*-intercept) of the function's graph
- any two points of the function's graph


Determining a linear equation will enable you to use the line of best fit to predict more accurately values based on trends.

THE EQUATION OF A LINE OF BEST FIT

A scatterplot may show readings related by a linear function; the data lies exactly on a line. Such an example is the relation of Fahrenheit readings to Celsius readings.

A scatterplot may also show data only **approximately linear**; that is, it is close to a line rather than lying exactly on it. An example of approximately linear data was shown in the graph of the points scored in hockey.

You can use a pencil-and-paper technique to analyse approximately linear data. Determine the linear equation corresponding to the data by using these steps:

Step1: Make a scatterplot of the data.

Step 2: Place a line of best fit for the data. Use a transparent ruler to draw a line among the points so that roughly half the points are on each side of the line.

Step 3: Determine the equation for the line of best fit using two points on the line of best fit.

Work through the following example.

Example

Karen, who coached basketball, reviewed the performance of her players over a ten-game period.

Karen collected this data.

Player	Average Time Played per Game (min)	Average Number of Points Made per Game
Armdal	17	14
Bouvier	16	11
Deen	12	6
House	20	20
Koupax	10	6
Leader	20	11
Muldoon	4	3
Naidoo	27	18
Rav	24	18
Reusel	12	10
Ryning	29	12
Smithson	17	8
Wetland	29	22
Wong	22	14
Zapisocki	7	4

a. Using pencil and paper, make a scatterplot of the data. Use the y-axis for average points and the x-axis for the average time.

b. Draw an estimated line of best fit for the graph.

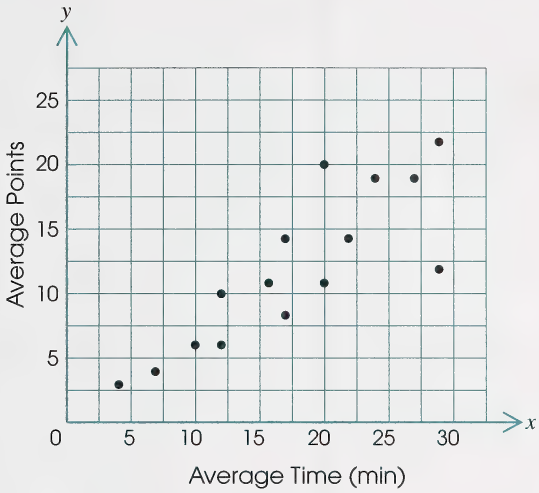
c. Calculate the slope of the line of best fit.

d. Write the equation for the line of best fit.

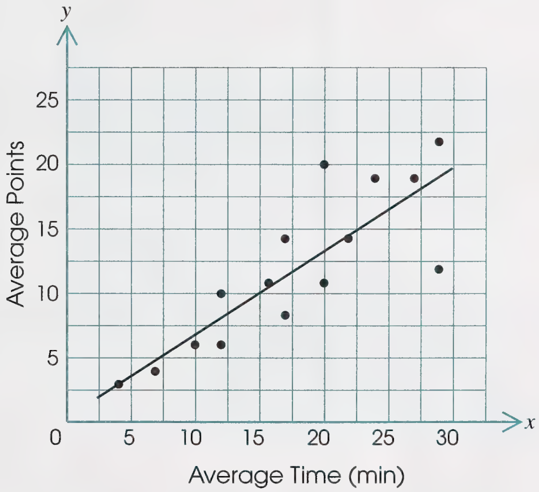
e. Suppose a player played 25 min in a game. How many points would you expect her to score based on the line of best fit?

Solution

a. The scatterplot will look like the following.

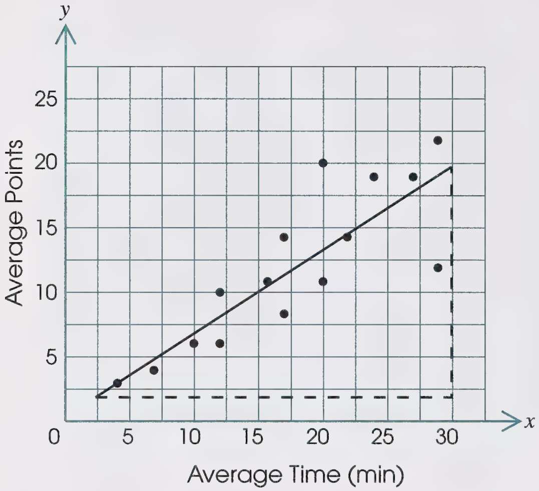


b. An estimated line of best fit will be similar to the following.



The line is placed so that about half the points are on each side of the line and the line follows the trend of the pattern of the points.

c. Draw a right triangle using the estimated line of best fit. The height of the triangle is the rise and the base is the run.



Draw dotted lines to complete a right triangle from two points where the line of best fit comes close to intersecting grid lines. For greatest accuracy, try to complete the largest triangle possible.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &\doteq \frac{19-2}{30-2} \\ &\doteq \frac{17}{28} \\ &\doteq 0.607 \end{aligned}$$

$$m = \frac{\text{rise}}{\text{run}}$$

From the graph, one possible point is $(30, 19)$ and another is $(2, 2)$.

The slope is about 0.61.

d. Let x be the time and y be the average points. Use a point **on** the line of best fit, for example, $(30, 19)$. Use the standard equation of a linear function and substitute the values of the point.

$$\begin{aligned} y &= mx + b \\ 19 &\doteq (0.607)(30) + b \\ b &\doteq 19 - (0.607)(30) \\ &\doteq 0.79 \end{aligned}$$

← Substitute values of the chosen point.

← Solve for b.

The equation is $y \doteq 0.61x + 0.79$.

Note: The equation is based on visual readings made from the scatterplot and is subject to small errors. You could also obtain the value for b by extending the line of best fit through the y-axis.

e. For a playing time of 25 min, $x = 25$. Substitute this value into the equation of the line of best fit.

$$\begin{aligned} y &\doteq 0.61x + 0.79 \\ &\doteq 0.61(25) + 0.79 && \longleftarrow \text{Substitute for } x. \\ &\doteq 16.04 \\ &= 16 && \longleftarrow \text{Round to the nearest whole number. (You cannot have partial points.)} \end{aligned}$$

According to the trend shown by the line of best fit, the player would be expected to make about 16 points.

Note: The expected number of points can also be obtained visually from the graph. Use interpolation.

10. Turn to page 288 of the textbook and read through "Investigation 1: The Spring."

If you have access to a set of known masses, do the investigation and answer exercises 1 to 6.

If you do not have access to a set of known masses, use the data given in the table of the answer to textbook exercises 2 and 3 on page 123 of the Appendix in this module and answer exercises 4 to 6 of the investigation.

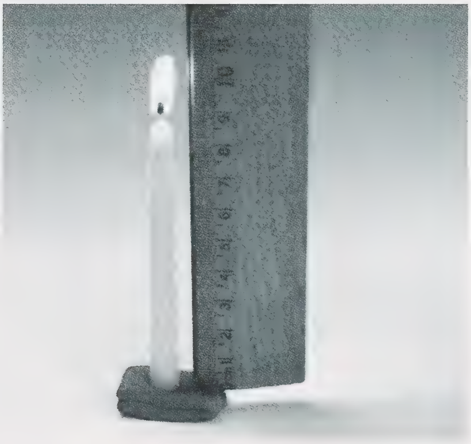
Compare your responses with the suggested answers in the Appendix, Activity 2, pages 123–124.

The line of best fit may have a negative slope. The next example illustrates a line of best fit with a negative slope.

11. Turn to page 289 of the textbook and answer exercises 1 to 6 of "Investigation 2: The Candle."

Note: You may use any small candle. A slender candle works best. You may use any modelling clay or playdough to hold the candle upright. You could also set up the candle in its wax on a jar lid or use a candle holder.

Caution: Never leave a burning candle unattended.



12. Turn to page 290 of the textbook and answer exercises 1, 3, and 4 of "Discussing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 125–127.

By graphing and placing the line of best fit, you can determine the linear equation for the trend. One factor affecting the accuracy of the equation is how well you place the line of best fit on the data. A method called the median-median method improves the placement of the line of best fit.

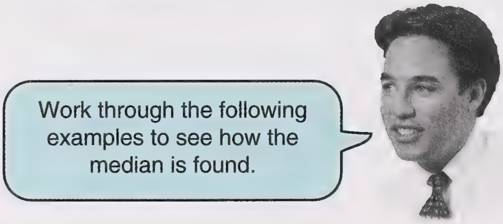
The Equation of a Median-Median Line of Best Fit

A multilane, divided highway has a median separating traffic going in opposite directions. The median of the highway is in the middle.

A median-median line of best fit more accurately represents the trend than an estimated line of best fit. In order to draw the median-median line of best fit, you need to be able to find the median of sets of data.

Finding the Median of a Set of Data

The **median** of a set of data is the middle value (or the mean of the middle values) of the set of values arranged in ascending order.


Work through the following examples to see how the median is found.

The set of data in this example has an odd number of values.

Example

In a field study, the number of petals on some flowers was found to be as follows: 18, 19, 16, 18, 19, 17, 16, 19, 18.

Determine the median of the data collected.

Solution

Step 1: Arrange the values in the set of data from least to greatest.

16, 16, 17, 18, 18, 18, 19, 19, 19

Step 2: There is an odd number of values; circle the middle value.

16, 16, 17, 18, ⑱, 18, 19, 19, 19

> There are 4 values to the left of the circled value and the same number on its right. This confirms that the circled value is the middle value.

The median number of petals is 18.

The set of data in the following example has an even number of values.

Example

In one hour, a clerk in a shoe store sold 8 pairs of shoes. The sizes of shoes sold were recorded by the clerk as he sold them.

The list of shoe sizes was as follows:

8, 9, 8, 7, 9, 11, 10, 8

Determine the median shoe size for this time period.

Solution

Step 1: Arrange the values in the set of data from least to greatest.

7, 8, 8, 8, 9, 9, 10, 11

Step 2: There is an even number of values in the set of data; place a line segment between the two middle values.

7, 8, 8, 8, | 9, 9, 10, 11

Step 3: Find the mean of the two values next to the line segment.

$$\frac{8+9}{2}=\frac{17}{2}$$
$$=8.5$$

The median shoe size is 8.5.

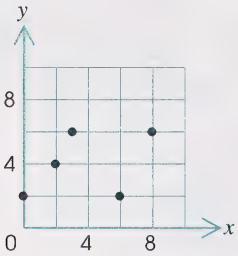
13. Determine the median for each set of data.

a. 2, 3, 3, 5, 9, 11, 15, 21, 22
b. 30, 20, 14, 21, 31
c. 10, 1, 3, 9, 3, 3, 7, 0, 1, 6, 8, 8

Compare your responses with the suggested answers in the Appendix, Activity 2, page 127.

Finding the Summary Point of a Set of Data

In determining the median-median line of best fit, you have to find the summary point of a set of points on the coordinate plane. The summary point of a set of points is similar to the median for a set of values.



Consider the given set of points.

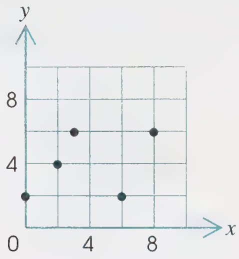
From the scatterplot, you cannot easily see which point(s) is(are) to be considered in the middle.

You may anticipate that the summary point should be an ordered pair rather than just a single value—but which ordered pair?

Work through the next example to see how the summary point is found.

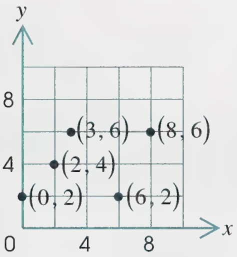
Example

Determine the summary point for the set of points on the scatterplot.



Solution

Step 1: Label the coordinates of the points on the graph. (This will identify the x-values and y-values.)



The summary point of a set of points is the point having the ordered pair (x_{med}, y_{med}), where x_{med} is the median of the x-values of the points of the set and y_{med} is the median of the y-values of the points.

Step 2: Arrange the x-values from least to greatest, and determine the median of the x-values.

0, 2, 3, 6, 8

The median of the x-values is 3.

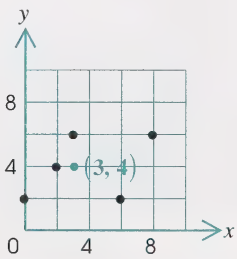
Step 3: Arrange the y-values from least to greatest, and determine the median of the y-values.

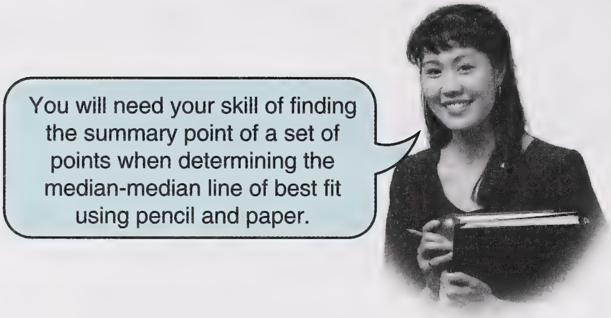
2, 2, 4, 6, 6

The median of the y-values is 4.

Step 4: Identify the summary point.

The median of the x-values is 3; the median of the y-values is 4. Therefore, the summary point of the set of points is $(3, 4)$.

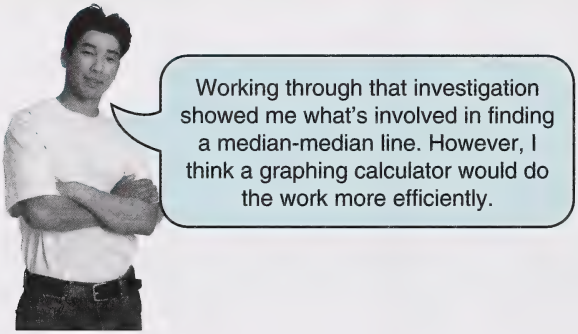



You will need your skill of finding the summary point of a set of points when determining the median-median line of best fit using pencil and paper.

Turn to page 294 of the textbook and read the first two paragraphs of Tutorial 6.7, "The Equation of a Median-Median Line of Best Fit."

14. Turn to pages 294 to 296 and answer exercises 1 to 9 of "Investigation: The Median-Median Line of Best Fit." **Note:** Use pencil and paper in this investigation.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 128–131.


Working through that investigation showed me what's involved in finding a median-median line. However, I think a graphing calculator would do the work more efficiently.

Turn to page 413 of the textbook and work through "Utility 14: Using the TI-83 to Find the Median-Median Line of Best Fit." (**Note:** Keep the data in your calculator. You will use it when working through the next example.)

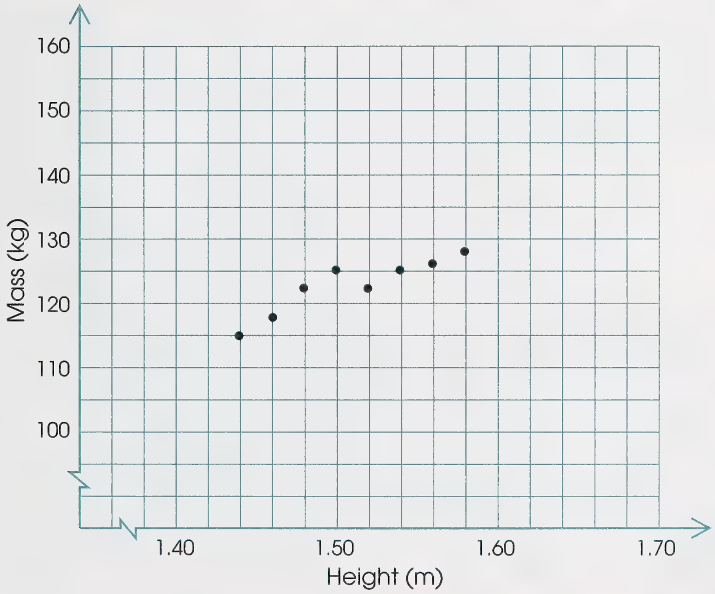
How does the line of best fit change when an outlier is added to a scatterplot? An outlier is a point that does not fit the general pattern of the other points of the scatterplot; an outlier lies away from the line of best fit.

Turn to page 296 of the textbook and work through "Example: Use technology to determine the median-median line of best fit for a data set." Since you have your data entered from the previous exercise, you need only add the coordinates of the extra point to your lists L1 and L2. Then, follow steps 3 and 4 of Utility 14 to get your new values for slope, the *y*-intercept, and the new line of best fit.

15. Turn to page 297 of the textbook and answer exercise 3 of “Discussing the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 2, page 132.

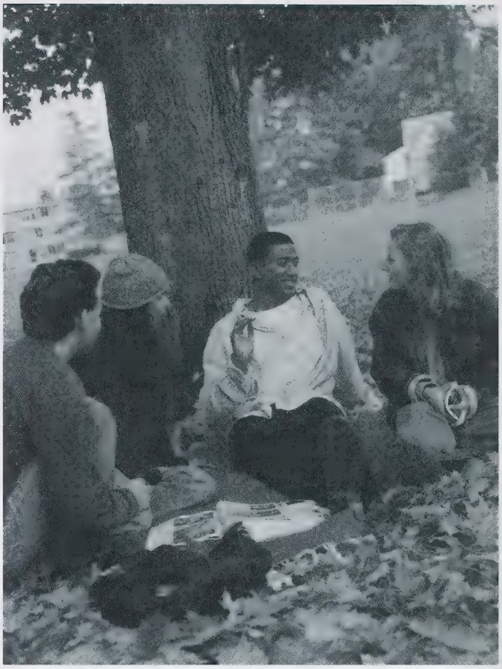
When graphing points on a grid, sometimes you may need to make your scales on the axes such that they are not continuous. You can use a zig-zag mark to indicate that there is a break in the scale. Study the following graph.



16. Turn to pages 297 and 298 of the textbook and answer exercises 1 and 2 of “Exercises: Checking Your Skills.” (**Note:** Use your graphing calculator for exercises 1.b. and 2.a.)

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 133–137.

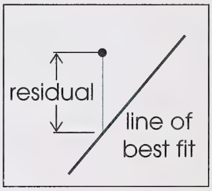
THE EQUATION OF A LINE OF BEST FIT BASED ON THE METHOD OF LEAST SQUARES



A group of students wondered how their science marks related to their mathematics marks. They produced a scatterplot of their final test results for the two subjects. For their group, those who did better on the math test also did better on the science test. How could the students best represent the relationship between the scores in the two subjects?

An estimated line of best fit, which is placed manually on a scatterplot, can give you a pretty good idea about the trend of the data. The median-median line of best fit is better. An often more dependable line of best fit is one based on the method of least squares. The method of least squares places the line so that the sum of the vertical distances to the points is the smallest.

In the method of least squares, the line of best fit is placed so that the sum of the squares of the **residuals** is minimized. The residual of a point is the vertical distance from the point to the line of best fit. (**Note:** The residual is not the shortest distance to the line.) The calculations for this line of best fit are so involved that they are best left for a calculator or computer to do.



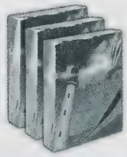
Turn to page 300 of the textbook and read Tutorial 6.8, "The Equation of a Line of Best Fit Using the Least Squares Method and Technology."

17. Refer to the scatterplot on page 300 of the textbook.

a. Which point has the largest residual? What is the value of this residual?

b. If the line of best fit were to be moved towards $(63, 29)$, for which points would the squares of the residuals increase? For which points would the squares of the residuals decrease?

Compare your responses with the suggested answers in the Appendix, Activity 2, page 137.



Work through the following example, based on the data on page 300 of the textbook, to see how a graphing calculator can be used to place the line of best fit according to the method of least squares.

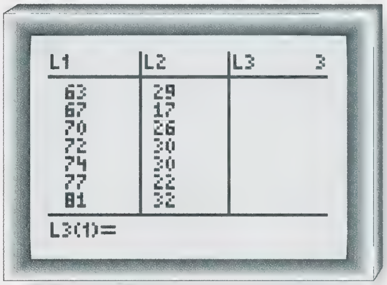
Example

Produce the scatterplot and line of best fit based on the method of least squares for the data of course and entrance marks (shown on page 300 of the textbook).

Solution

Step 1: Press STAT 1 or STAT ENTER to display the list editor. If necessary, clear the lists.

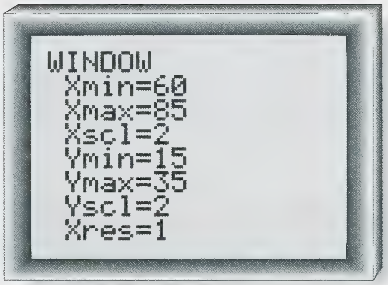
Enter the course marks in list L1. Enter the entrance marks in list L2.



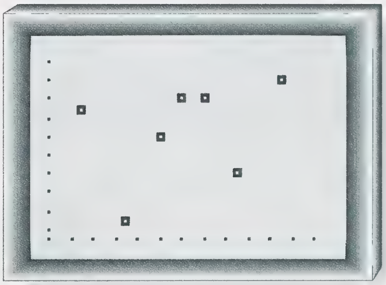
```
L1     |L2     |L3      3
-------------------------
 63    | 29    |
 67    | 17    |
 70    | 26    |
 72    | 30    |
 74    | 30    |
 77    | 22    |
 81    | 32    |
-------------------------
L3(1)=
```

Step 2: Set appropriate WINDOW settings; one example is given here.

Press GRAPH to display the scatterplot.



```
WINDOW
 Xmin=60
 Xmax=85
 Xscl=2
 Ymin=15
 Ymax=35
 Yscl=2
 Xres=1
```

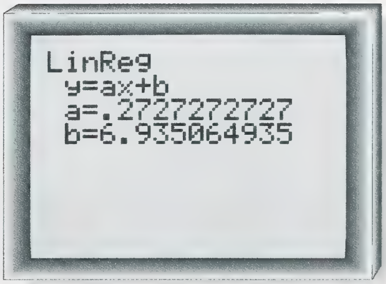


Step 3: Determine the equation of the line of best fit based on the method of least squares.

The right arrow key highlights CALC.

Press STAT ▶ 4 ENTER.

The 4 key chooses LinReg(ax + b).



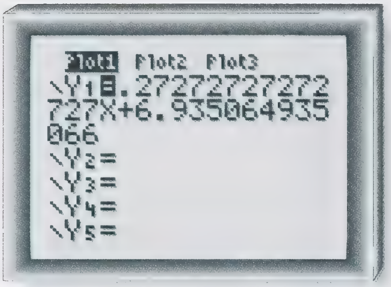
```
LinReg
 y=ax+b
 a=.2727272727
 b=6.935064935
```

The equation of the line of best fit is about $y = 0.27x + 6.9$.

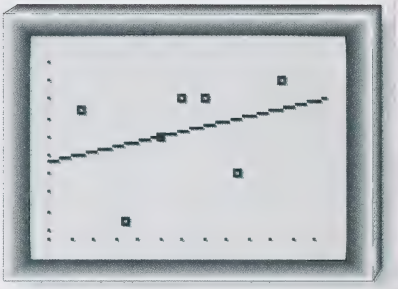
Step 4: Graph the line of best fit. Place the equation of best fit into the Y= editor. Press Y=. If necessary, clear any functions remaining from previous work.

Place the cursor in the "Y1=" line.

Press VARS 5 ▶ ▶ 1 GRAPH.



```
Plot1 Plot2 Plot3
\Y1=.27272727272
727X+6.935064935
066
\Y2=
\Y3=
\Y4=
\Y5=
```



Now it's your turn to produce a line of best fit using the least squares method. Turn to page 301 of the textbook and work through exercises 1 to 6 of "Investigation 1: Least Squares Method for Finding the Equation of the Line of Best Fit." **Note:** In exercise 5, press STAT, not 2nd [STAT PLOT].

18. Turn to page 302 of the textbook and answer exercises 1 to 5 of "Investigation 2: Compare the Lines of Best Fit Obtained Using Different Methods."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 137–140.

Recall that the mean and median of a set of values are both averages. The mean may often be the best kind of average. However, the mean is susceptible to disturbance from an outlier whereas the median is more stable.

19. Find the mean and median of each of the following sets of data.

a. 27, 25, 27, 27, 30, 26

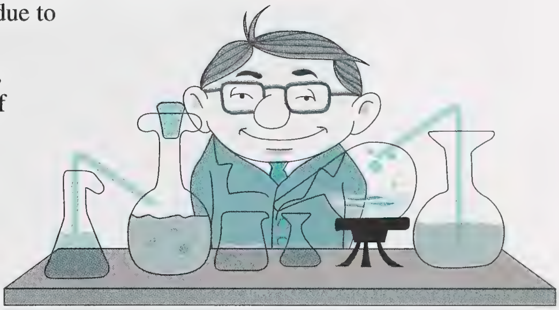
b. 51, 25, 27, 27, 30, 26

c. 63, 25, 27, 27, 30, 26

d. 99, 25, 27, 27, 30, 26

Compare your responses with the suggested answers in the Appendix, Activity 2, page 140.

An outlier in a set of data may be due to experimental error or atypical (extraordinary) circumstances and, therefore, it is not representative of the trend. What is the effect of an outlier on the placement of the line of best fit? The method of determining the line of best fit that is least affected by such an outlier is the method that is often the most appropriate for the data.



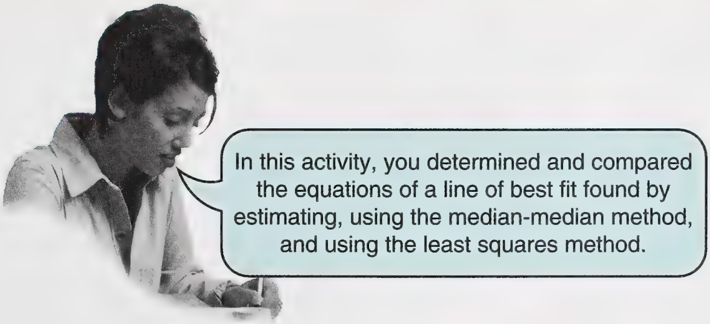
The data in Investigation 2 (page 302 of the textbook) contains an outlier. Which method of finding a line of best fit is most appropriate for such data?

20. Turn to page 302 of the textbook and answer exercise 6 of "Investigation 2: Compare the Lines of Best Fit Obtained Using Different Methods."

21. Turn to page 303 of the textbook and answer exercises 3, 4, and 6 of "Discussing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 140–141.

LOOKING BACK



In this activity, you determined and compared the equations of a line of best fit found by estimating, using the median-median method, and using the least squares method.

22. Turn to page 303 of the textbook and answer "Communicating the Ideas."

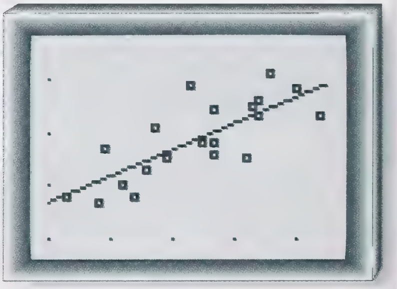
Compare your response with the suggested answer in the Appendix, Activity 2, page 141.

ACTIVITY 3

The Correlation Coefficient

Have you ever had a blood-pressure test from your doctor? For a medical checkup, your doctor will take your blood pressure when you are rested and relaxed. Recent exertion will cause a higher reading.

In special cases, doctors may want to know more about how your body is affected by physical stress. Your oxygen uptake, pulse rate, blood pressure, and other measurements will be taken while undergoing exercise—often on a treadmill. The stress can be increased by increasing the incline and speed of the treadmill.

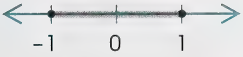


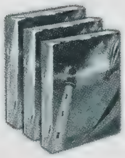
The given graph shows there is some connection between blood pressure and stress.

The data seems to follow the line of best fit on the scatterplot; but the points are some distance away from the line. The data is appproximately linear. Statisticians have devised a measure to indicate how closely data lies to a line of best fit—the correlation coefficient.

Turn to page 304 of the textbook and read Tutorial 6.9, “The Correlation Coefficient.”

The **correlation coefficient** for a set of data points is a number, *r*, indicating how well the set can be represented by a linear function. The value of *r* is always equal to or between -1 and $+1$. The values can be represented as $-1 \leq r \leq 1$. The values can also be shown on a number line.





1. Turn to pages 305 and 306 of the textbook and answer exercises 1.a., d., f., g., i., and j. of "Investigation: The Correlation Coefficient." Draw the scatterplots by hand on graph paper. Then, use your graphing calculator to find the value of *r*.
   Note: Before you enter your data and set the WINDOW, remember to turn Diagnostic On, as instructed on page 304 of your textbook.

   Then, for each of the scatterplots, answer exercises 2 and 3. **Note:** If you can work as a group with others, divide the work as indicated in the textbook and graph all the data in exercise 1.

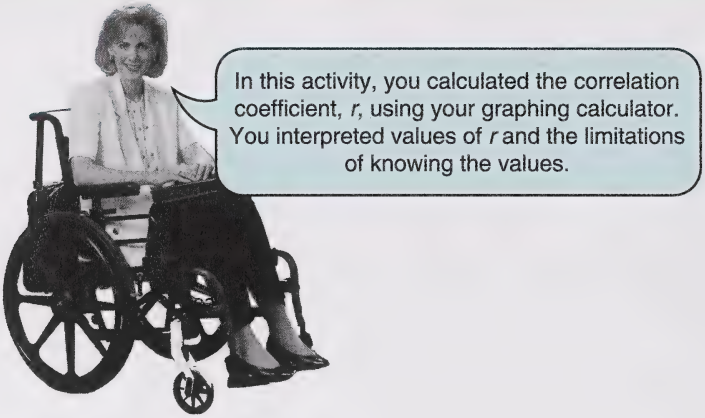
2. Turn to page 306 of the textbook and answer exercises 5 and 6 of "Investigation: The Correlation Coefficient."

3. Turn to page 306 of the textbook and answer exercises 1 to 5 of "Discussing the Ideas."

4. Turn to pages 306 to 308 of the textbook and answer exercises 1.a., 1.d., 2, 4, and 5 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 142–150.

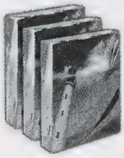
LOOKING BACK


In this activity, you calculated the correlation coefficient, r, using your graphing calculator. You interpreted values of r and the limitations of knowing the values.

5. Give the meaning of the term *correlation coefficient*. Write a paragraph or two to explain what various values of r represent.

Compare your response with the suggested answer in the Appendix, Activity 3, page 150.

Follow-up Activities



This module dealt with Chapter 6: Linear Functions.

Turn to page 312 of the textbook and read the chart in "What Do I Need To Know?" Notice the skills and concepts that were developed in this module. Also, read the important results and formulas that you discovered.

1. Turn to pages 314 to 316 of the textbook and answer exercises 7.b., 8.b., 10, and 12 of Part B of "What Should I Be Able To Do?"

2. Complete the following chart so that the data has a correlation coefficient of -1. Choose your own method to solve this problem.

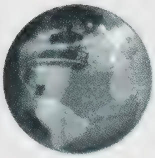
x	2	4	5
y	11		2

Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 151–156.

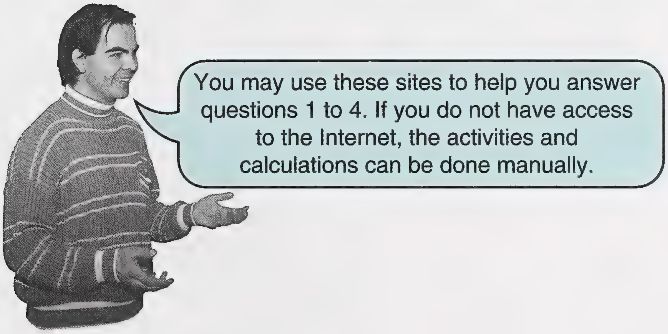
If you had difficulties understanding the skills and concepts in Module 6: Linear Functions, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

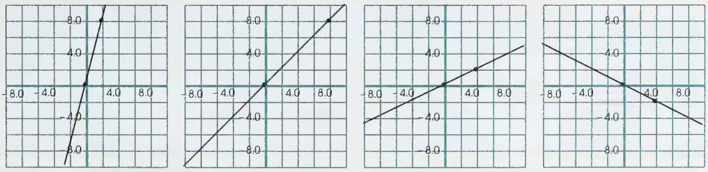
In this module, you saw how the values of *m* and *b* in the linear equation $y = mx + b$ relate to the slope and *y*-intercept. If possible, go to one of the following Internet sites for an opportunity to investigate this connection further:



- **http://www.exploremath.com/activities/Activity_page.cfm?ActivityID=16**
- **http://id.mind.net/~zona/mmts/functionInstitute/linearFunctions/lsif.html**


You may use these sites to help you answer questions 1 to 4. If you do not have access to the Internet, the activities and calculations can be done manually.

1. Refer to the following series of graphs. The graph is rotated around the origin. Note how *m* and *b* change. Record the values in the chart.



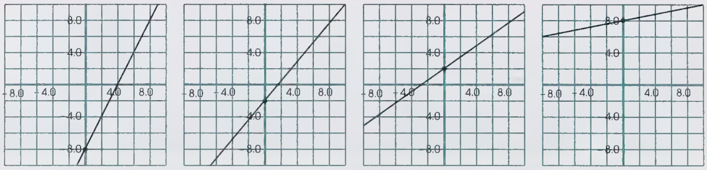
m				
b				

   a. Describe how the slope change in the series of graphs.

   b. How does the value of *m* change?

   c. Indicate how the value of *b* is affected.

2. Refer to the following series of graphs. The graph is shifted up or down on the y-intercept. Note how m and b change. Record the values in the chart.



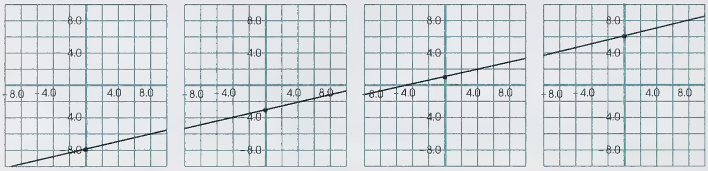
m				
b				

a. Describe how the slope and y-intercept change in the series of graphs.

b. How does the value of m change?

c. Indicate how the value of b is affected.

3. Refer to the following series of graphs. The graph is shifted on the y-intercept. Note how m and b change. Record the values in the chart.



m				
b				

a. Describe how the slope and y-intercept change in the series of graphs.

b. How does the value of m change?

c. Indicate how the value of b is affected.

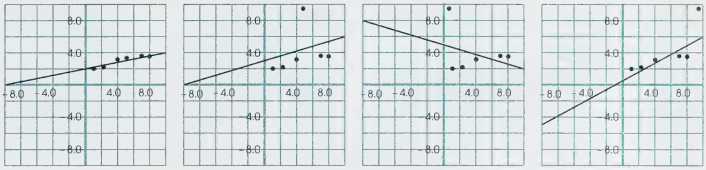
4. Based on your investigations for questions 1, 2, and 3 (and any additional investigations you did on your own) indicate what characteristic of a line graph correspond to m and b in the equation $y = mx + b$.

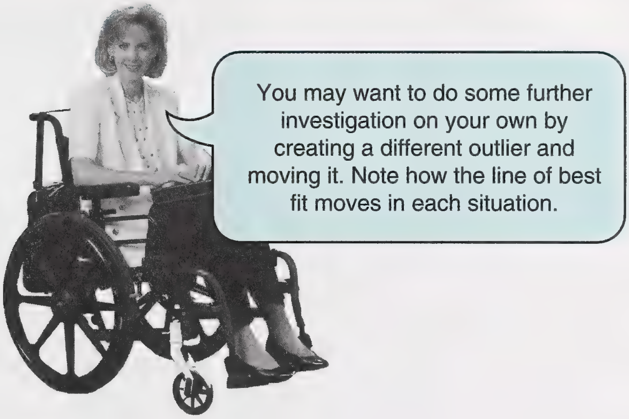
Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, pages 157–158.

In this module, you determined the line of best fit based on the method of least squares.

Refer to the following series of scatterplots. The last three graphs each have an outlier in a different position.

A line of best fit for each graph is drawn using the method of least squares. Note the change in position of the line of best fit.




You may want to do some further investigation on your own by creating a different outlier and moving it. Note how the line of best fit moves in each situation.

This site provides a model to help visualize how outliers affect the line of best fit:

http://www.exploremath.com/activities/Activity_page.cfm?ActivityID=6

If you do not have access to the Internet, continue your investigations using your graphing calculator.

5. Based on your investigations, generally, state how a line of best fit changes by having an outlier appear in these positions:

- **a.** above the middle of a set of points
- **b.** above and to the left of the middle of a set of points
- **c.** above and to the right of the middle of a set of points
- **d.** below the middle of a set of points

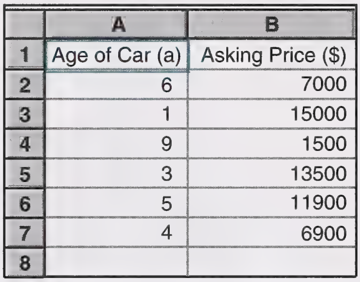
Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, page 158.

ENRICHMENT

In this module, you used a graphing calculator to produce a scatterplot and a line of best fit and to determine the correlation coefficient. Alternatively, you can also use a spreadsheet program.

Work through these steps to create a scatterplot and line of best fit using a spreadsheet program.

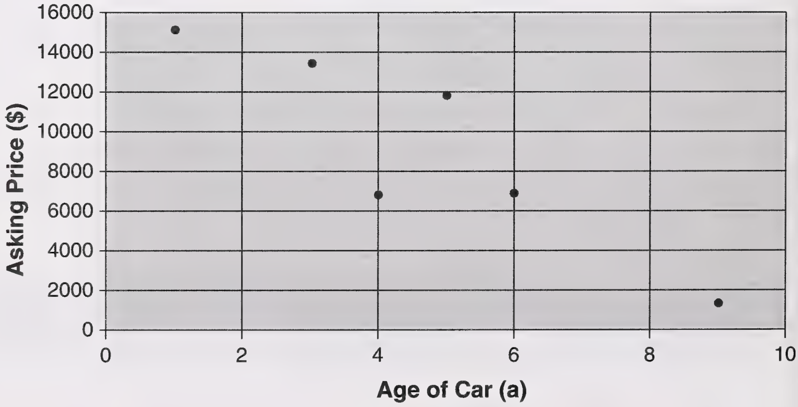
Step 1: Enter the data into the spreadsheet. (This example uses data from textbook exercise 12 on page 316.) Then, select cell A1. (Alternatively, select cells A1 through B7.)



	A	B
1	Age of Car (a)	Asking Price ($)
2	6	7000
3	1	15000
4	9	1500
5	3	13500
6	5	11900
7	4	6900
8		

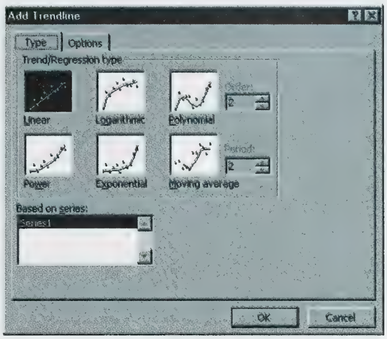
Step 2: Click on the Chart Wizard icon in the toolbar. A Chart Wizard screen pops up. Select **XY (Scatter)** from the Chart type and from the Chart sub-type. Click **Next**.

Step 3: Select various Chart options by clicking **Next**. (You should be on the page displaying Step 3 of 4 – Chart Options.) When you are satisfied with your finished graph, click on the Finish button. The scatterplot appears. One example is shown here.

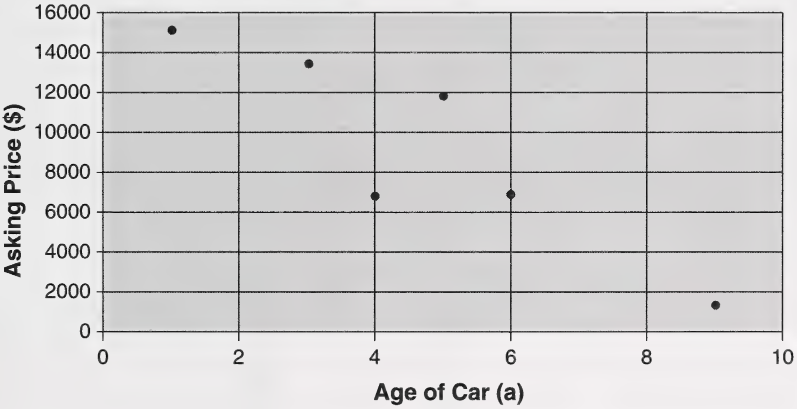


Step 4: Choose **Add Trendline...** from the Chart pull-down menu.

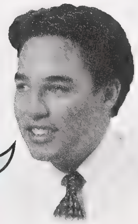
The Add Trendline window appears. Select **Linear**.



Step 5: Click **OK**. The line of best fit appears on the scatterplot.
Note: Save this spreadsheet file.

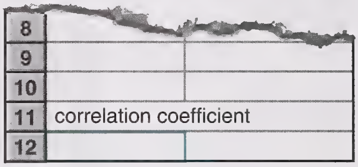


The spreadsheet is a powerful tool for analysing data. You may wish to use a spreadsheet application to re-do some of the analysing you did earlier with your graphing calculator.



You can also determine the correlation coefficient for the data by using a spreadsheet program. Work through the following steps.

Step 1: Enter the words **correlation coefficient** in an empty cell of the spreadsheet and then select the cell below the text.



8		
9		
10		
11	correlation coefficient	
12		

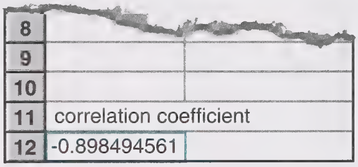
Step 2: Select ***fx* Function...** from the pull-down Insert Menu. A Paste Function screen will appear. Select **Statistical** from the Function category and **CORREL** from the Function name. Press **OK**. A CORREL screen will appear.

Step 3: With the cursor in Array 1 position in the panel, drag across the figures in the first column of the data. (You may need to drag your CORREL screen to a different location to see your numbers.) Press **Tab** to move the cursor to the Array 2 position of the panel.

With the cursor in Array 2 position of the panel, drag across the figures in the second column of the data.

Note: Alternatively, you could type in the cell number of the first entry in the column, then a colon, and then the cell number of the last entry in the column.

Step 4: Press **OK**. The correlation coefficient appears in the selected cell.



8		
9		
10		
11	correlation coefficient	
12	-0.898494561	

Use your spreadsheet file to answer the following question.

Select the cells for the data point $(4, 6900)$. Press **Delete**. Describe and explain what happens to the scatterplot, the line of best fit, and the value of the correlation coefficient when you remove the data. (Press **Ctrl Z** to undo the change.)

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, pages 158–159.

MODULE PROJECT

CONTRASTING TWO SPECIES

Completing the Project

You should now have done most of the research for the Module 6 project, Contrasting Two Species. Your research notes should be in the project section of your mathematics binder.

In this module, you investigated the characteristics of a gorilla to determine aspects of its form. Then, you inferred connections between the animal's form and its function. In order to make connections between form and function, you needed to know how and where gorillas live. You also collected data on humans to have a standard for comparison. Part of your project involved the investigation of the relation between height and arm span for humans and gorillas.

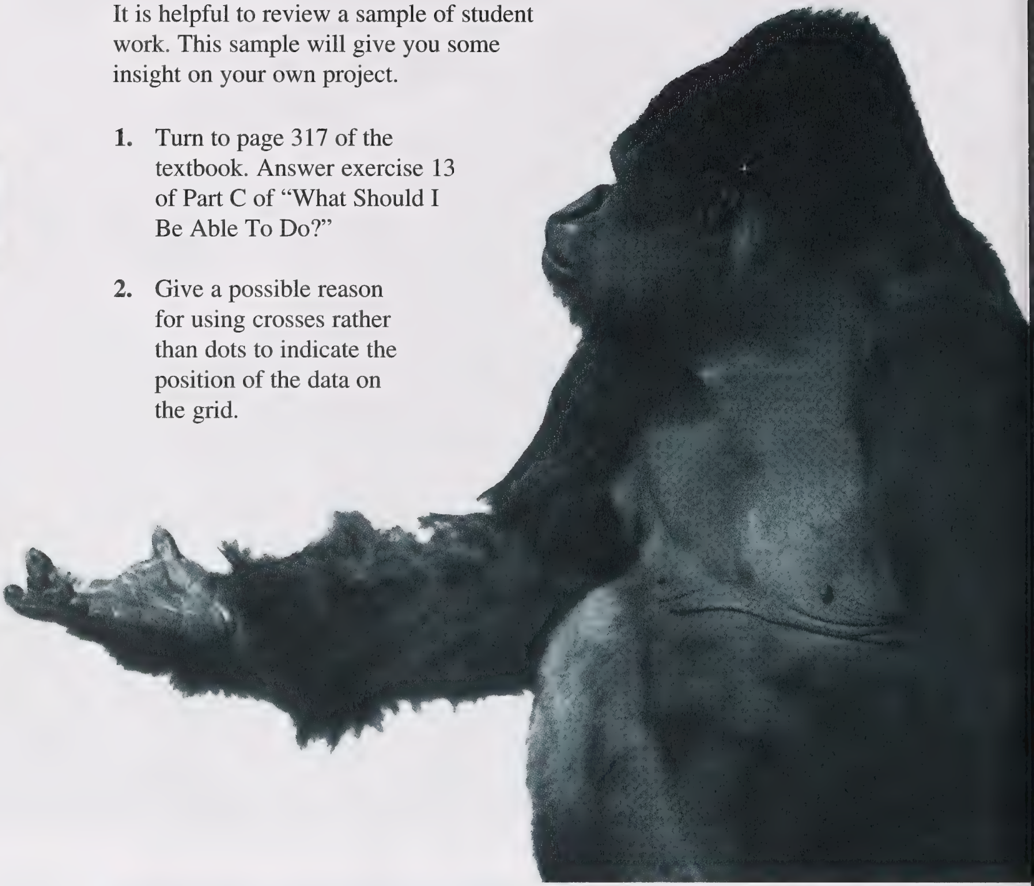
Turn to page 292 of the textbook and read "Height/Arm-Span Relationship."

Follow the directions in the bulleted list. Place your work in the project section of your mathematics binder.

It is helpful to review a sample of student work. This sample will give you some insight on your own project.

1. Turn to page 317 of the textbook. Answer exercise 13 of Part C of "What Should I Be Able To Do?"

2. Give a possible reason for using crosses rather than dots to indicate the position of the data on the grid.



Compare your responses with the suggested answers in the Appendix, Module Project, page 159.

Module Project

Remember, your project will be marked on the accuracy of information, the mathematical representation of data, the clarity of language, and the validity of inferences. Focus on the measurement data associated with gorillas and humans. Demonstrate your understanding of scatterplots and linear functions.

When you are satisfied with the information and analysis of your data, turn to your Project Booklet and complete the Module Project: Contrasting Two Species.

Submit your completed Module 6 Project Booklet to your teacher.

Module Summary

In this module, you graphed functions and applied linear functions to direct variations, arithmetic sequences, and other contexts. You determined the equation of a line and the equation of an estimated line of best fit. You also found the median-median line of best fit and the line of best fit based on the method of least squares. Finally, you determined the correlation coefficient for data and interpreted the value of the correlation coefficient to indicate the how well the data formed a line.

Measuring and collecting data and analysing the data allows predictions to be made. Predictions based on linear relations can be accurate even for extra-terrestrial environments, as you learned from the discovery of dust deposition by the Mars Pathfinder mission.

Dust settling could threaten the functioning of solar panels on the long missions to Mars or if the missions ran during the dust-storm season, which occurs when Mars is closest to the Sun. It will be projections based on data analysis that will help in planning future visits to Mars or other planets.

NASA

Module Assignment

To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.

Submit your completed Module 6 Assignment Booklet to your teacher.

Applied Mathematics 10

APPENDIX

GLOSSARY

Arithmetic sequence: a sequence in which the difference between one term and the next is always the same

Correlation coefficient: a number indicating how well the data can be approximated by a linear function

Dependent variable: the output variable of a function

Direct variation: a function having a y-intercept of zero; a relation in which the input and output variables are always in the same ratio

Domain: all the x-values of the points of the graph of the function

Empirical data: data that comes from observation or measurement, rather than from a rule or a theoretical model

Function: a relation in which each input value has only one output value

Independent variable: the input variable of a function

Linear function: a function whose equation can be written in the form $y = mx + b$ and whose graph is a line or points of a line

Median: the middle value or mean of the middle values of a set of data arranged in ascending order

Natural numbers: the set of numbers 1, 2, 3, …

Outlier: a point on a graph that lies away from the line of best fit

Range: all the y-values of the points of the graph of the function

Residual (of a point): the vertical distance of a point from the line of best fit

Scatterplot: a graph consisting of individual points whose coordinates represent values of an independent and a dependent variable

Sequence: an ordered set of numbers

Slope of a line: the ratio $\frac{(y_2 - y_1)}{(x_2 - x_1)}$, where (x_1, y_1) and (x_2, y_2) are points on the line

Summary point: the point having the ordered pair (x_{med}, y_{med}), in which x_{med} is the median of the x-values of all the points of the set and y_{med} is the median of the y-values of all the points of the set

Term: a number belonging to a sequence

***x*-intercept:** the x-coordinate of the intersection point of a graph and the horizontal axis

***y*-intercept:** the y-coordinate of the intersection point of a graph and the vertical axis

SUGGESTED ANSWERS

Activity 1: Linear Functions

1. Textbook exercises 2 and 4 of "Practise Your Prior Skills," p. 257

2. $5 - y = 4 + 3x$

$-y = 4 + 3x - 5$ ← Subtract 5 from both sides.

$-y = -1 + 3x$ ← Simplify.

$y = -3x + 1$ ← Divide both sides by -1, the coefficient of y.

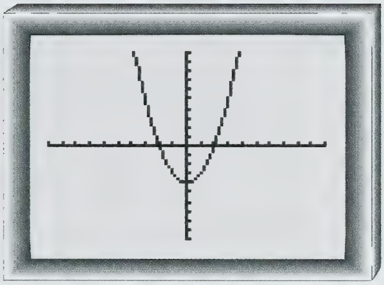
4. $2 - 4y = 6x$

$-4y = 6x - 2$ ← Subtract 2 from both sides.

$y = -\frac{3}{2}x + \frac{1}{2}$ ← Divide both sides by -4, the coefficient of y.

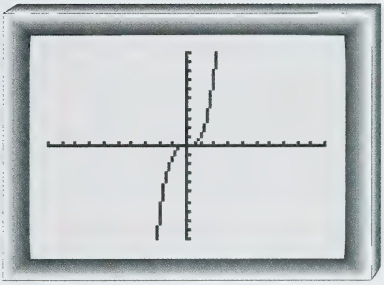
2. a. Textbook exercise 1 of "Investigation 2: Linear Functions," p. 257

1. The graph of $x^2 - 4$ is as follows.



The graph of $x^2 - 4$ does not represent a linear function; it is not a straight line.

The graph of $y = x^3$ is as follows.

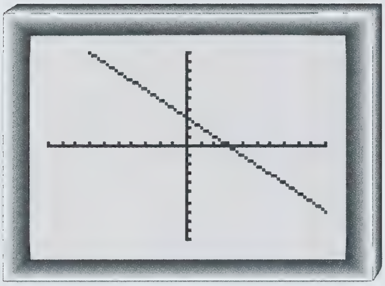


The graph of $y = x^3$ does not represent a linear function; it is not a straight line.

Activity 1 (continued)

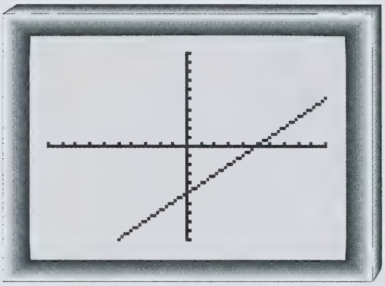
Rearrange the equation $y+5=x$ into y-intercept form: $y=x-5$.

The graph of $y=-x+3$ is as follows.



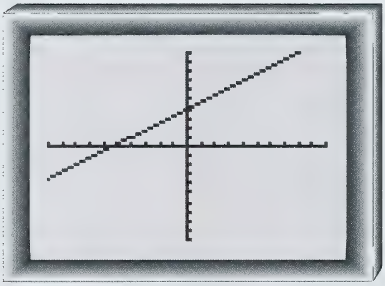
The graph of $y=-x+3$ represents a linear function; it is a straight line.

The graph of $y=x-5$ is as follows.



The graph of $y=x-5$ represents a linear function; it is a straight line.

The graph of $y=0.75x+4$ is as follows.



The graph of $y=0.75x+4$ represents a linear function; it is a straight line.

b. These are some of the similarities between the equations that represented straight lines:

- The equations do not have x to a power less than zero nor greater than 1.
- There is only one variable in each term.
- There are no radicals.
- The equations can expressed in the form $y=mx+b$, where m and b stand for constant values.

3. **a.** $y=-4x+3$

$=-4x+3$

$m=-4; b=3$

b. $y+5=3x$

$y=3x-5$

$=3x+(-5)$

$m=; b=-5$

c. $y=0.25x-1$

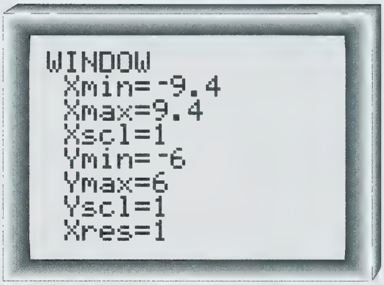
$=0.25x+(-1)$

$m=0.25; b=-1$

4. **Textbook exercises 1 to 4 of "Investigation 3: Find the Intercepts," p. 258**

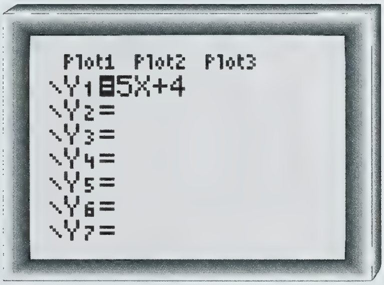
Note: Use a "friendly" WINDOW setting for these exercises.

Press WINDOW and enter the given values.

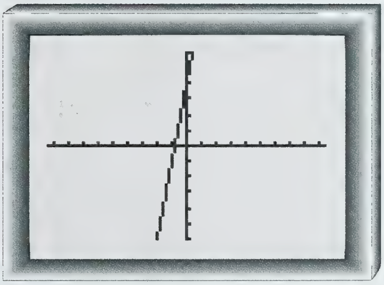


```
WINDOW
 Xmin=-9.4
 Xmax=9.4
 Xscl=1
 Ymin=-6
 Ymax=6
 Yscl=1
 Xres=1
```

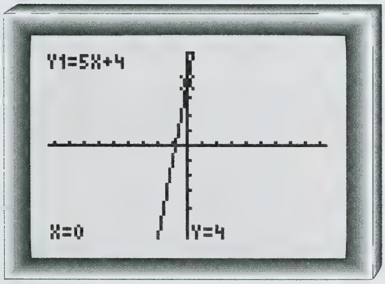
1. The graph of $y=5x+4$ is as follows.



```
Plot1 Plot2 Plot3
\Y1=5X+4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



2. Press TRACE. Use the arrow keys, if necessary, to have the flashing cursor show the point of intersection when $x=0$.

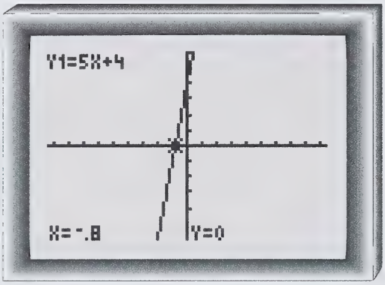


```
Y1=5X+4
X=0          Y=4
```

The coordinates of the point of intersection with the y-axis are $(0, 4)$.

Activity 1 (continued)

3. Use the arrow keys to find the point of intersection when $y = 0$.

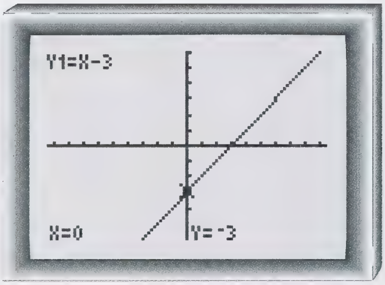
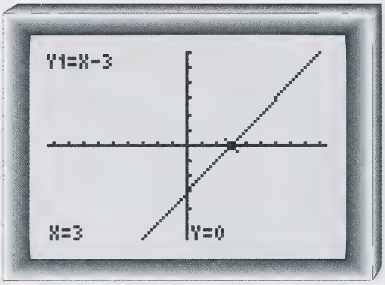
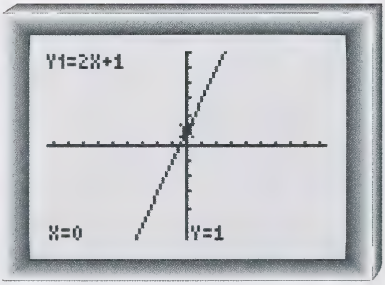
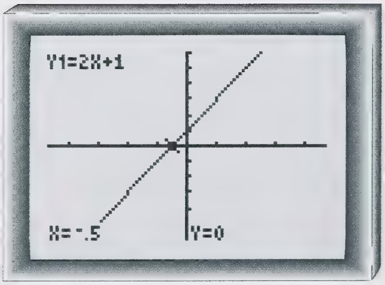


Y1=5X+4

X=-.8 Y=0

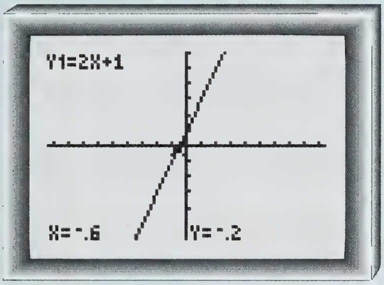
The coordinates of the point of intersection with the x-axis are $(-0.8, 0)$.

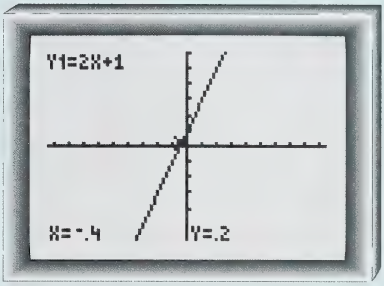
4.

Equation	Crossing the *y*-axis	Crossing the *x*-axis
$y = x - 3$	 Y1=X-3 X=0 Y=-3 The point of intersection is $(0, -3)$.	 Y1=X-3 X=3 Y=0 The point of intersection is $(3, 0)$.
$y - 2x = 1$ $y = 2x + 1$ $\uparrow$ Put the equation in "$y =$" form.	 Y1=2X+1 X=0 Y=1 The point of intersection is $(0, 1)$.	 Y1=2X+1 X=-.5 Y=0 The point of intersection is $(-0.5, 0)$. (See the following notes.)

Note: The current WINDOW settings do not allow the TRACE cursor to land on the value of $y = 0$.

As indicated earlier, setting the WINDOW to $\text{Xmin} = -9.4$ and $\text{Xmax} = 9.4$ makes the TRACE cursor move in *x*-intervals of 0.2; the resulting *y*-values for the equation jump over the intersection value of 0. One way of overcoming this problem is to reset the WINDOW settings to a different multiple of 94.



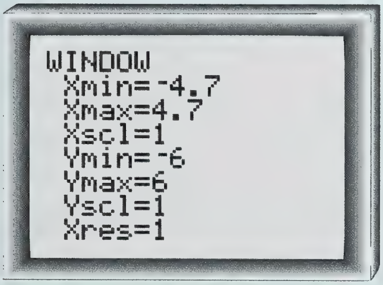


4.7 is a multiple of 9.4.

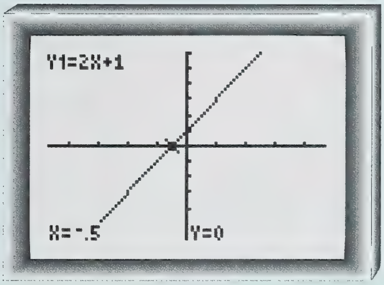
$0.5 \times 9.4 = 4.7$

If you adjust the WINDOW setting to $\text{Xmin} = -4.7$ and $\text{Xmax} = 4.7$, the TRACE cursor will move in *x*-intervals of 0.1, and then you may land on the intersection point with the *x*-axis.

Set your WINDOW to the values shown in the following screen. You should now be able to TRACE onto the point where the graph crosses the *x*-axis.

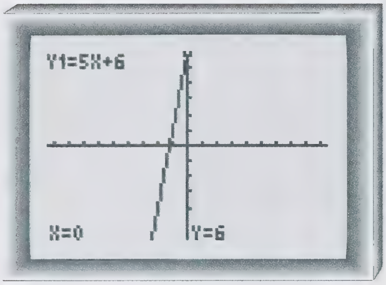
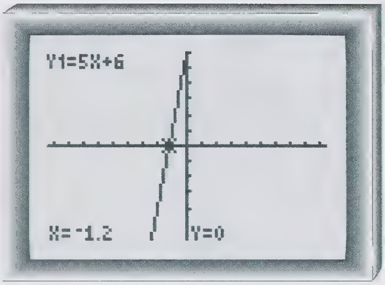
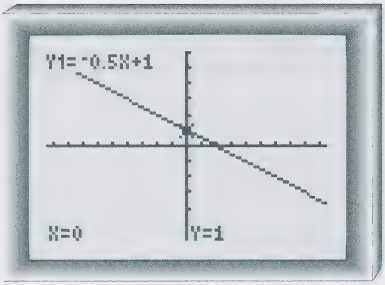
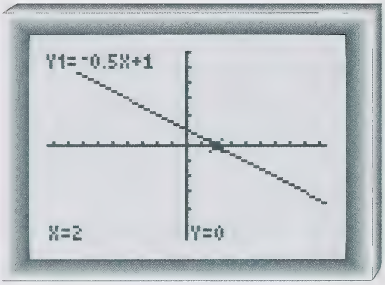
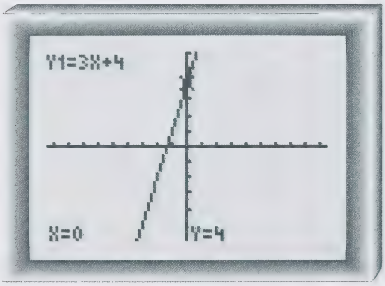
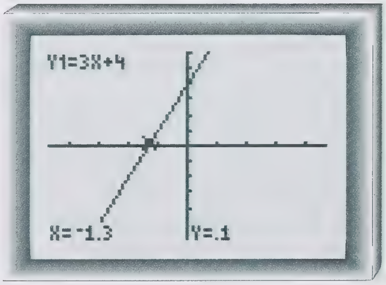


```
WINDOW
 Xmin=-4.7
 Xmax=4.7
 Xscl=1
 Ymin=-6
 Ymax=6
 Yscl=1
 Xres=1
```



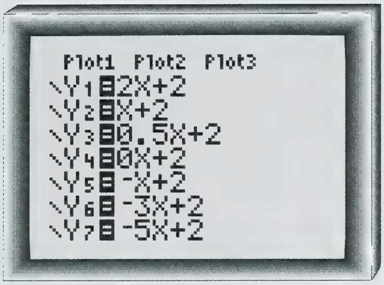
Note: At times, it is possible to not land on the intersection point with any "friendly" WINDOW. For example, in the trace for $y = 3x + 4$, which follows, the *x*-value of the intersection point is close to -1.3. You will soon learn about another way to find intersection points. Then, you will be able to find intersection points that the TRACE cursor "jumps" over.

Activity 1 (continued)

Equation	Crossing the *y*-axis	Crossing the *x*-axis
$y=5x+6$	 Y1=5X+6 X=0 Y=6 The point of intersection is $(0,6)$.	 Y1=5X+6 X=⁻1.2 Y=0 The point of intersection is $(-1.2,0)$.
$2y+x=2$ $y=-\frac{1}{2}x+1$ or $y=-0.5x+1$	 Y1=⁻0.5X+1 X=0 Y=1 The point of intersection is $(0,1)$.	 Y1=⁻0.5X+1 X=2 Y=0 The point of intersection is $(2,0)$.
$y=3x+4$	 Y1=3X+4 X=0 Y=4 The point of intersection is $(0,4)$.	 Y1=3X+4 X=⁻1.3 Y=.1 The closest TRACE cursor to the point of intersection is $(-1.3,-0.1)$. (See the preceding notes on "friendly" WINDOW settings.)

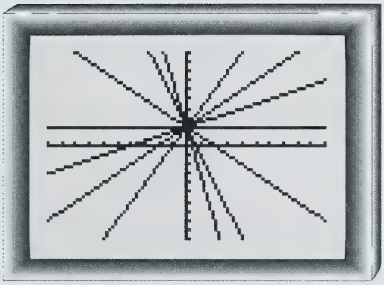
5. Textbook exercises 1 to 10 of “Investigation 1: Changing *m*,” p. 260

1. Your “ Y = ” list should look as follows.



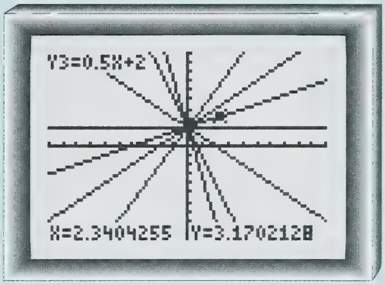
```
Plot1 Plot2 Plot3
\Y1=2X+2
\Y2=X+2
\Y3=0.5X+2
\Y4=0X+2
\Y5=-X+2
\Y6=-3X+2
\Y7=-5X+2
```

2. Your graphs should look like this.



Press ZOOM 6 WINDOW to obtain the standard WINDOW.

Note: In order to identify the individual graphs, use the TRACE feature. Press TRACE and move the cursor a little to the right of the y-axis. Then, use the ▲ and ▼ arrow keys to move the cursor onto different lines. The related equation will be displayed. See the example to the right.



```
Y3=0.5X+2
X=2.3404255  Y=3.1702128
```

Activity 1 (continued)

3. The graphs are all straight lines and cross the y-axis at the same place—at $(0, 2)$.

4. All the equations include the constant term 2.

5. The graphs have different slopes.

6. The coefficient of x differs in the equations.

7. The y-intercepts are all 2.

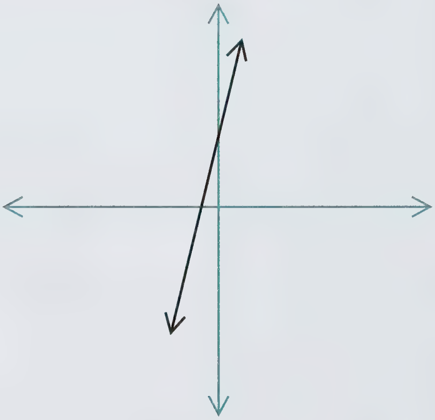
8. As m decreases, the graph rotates clockwise around the common intersection point with the y-axis.

   As m increases, the graph rotates counterclockwise around the common intersection point with the y-axis.

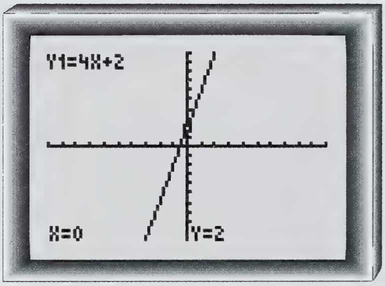
9. The variable m represents the slope.

10. Sketches may differ. An example follows. The calculator graph of $y = 4x + 2$ is also shown.

Sketch of Graph



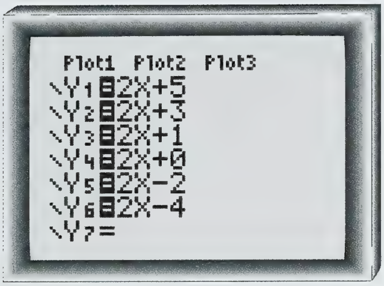
Calculator Screen



The graph of $y = 4x + 2$ crosses the y-axis at 2.

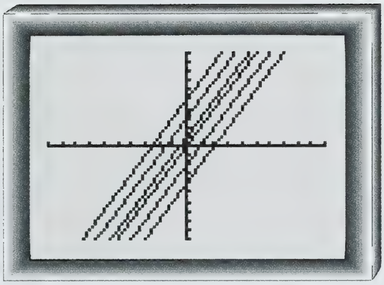
6. Textbook exercises 1 to 8 of "Investigation 2: Changing *b*," p. 261

1. The equations should be entered as shown.



```
Plot1 Plot2 Plot3
\Y1=2X+5
\Y2=2X+3
\Y3=2X+1
\Y4=2X+0
\Y5=2X-2
\Y6=2X-4
\Y7=
```

2. Your graphs should look like the following.



Press ZOOM 6 WINDOW to set the standard WINDOW.

Note: In order to identify the individual graphs, press TRACE. Then, press ▲ or ▼ ("up" and "down" arrow keys) to change the line or graph selection.

3. The graphs are parallel and have the same slope.

4. The coefficient of x in all the equations is the same; the coefficient is 2.

5. The x- and y-intercepts of the graphs are different.

6. The constant term, b, differs in each equation.

7. As b decreases , the graph moves in a negative direction of the vertical axis (that is, downwards).

   As b increases, the graph moves in the positive direction of the vertical axis (that is, upwards).

   The slope remains unchanged.

8. The value of b represents the y-intercept (the place where the graph crosses the y-axis).

7. $3x-4y+20=0$

$$3x+20=4y$$ ← Add 4 y to both sides.

$$4y=3x+20$$ ← Interchange LS and RS.

$$y=\frac{3}{4}x+5$$ ← Divide by 4 throughout.

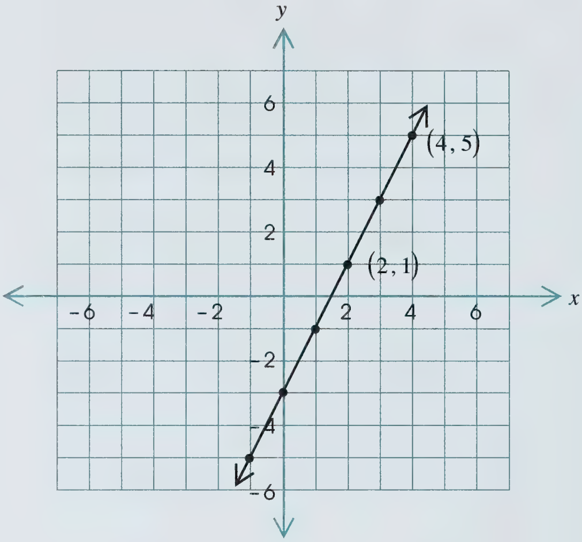
8. **a.** Pick any two points on the graph. The points you choose may differ.

$$(x_1, y_1)=(2, 1)$$
$$(x_2, y_2)=(4, 5)$$

Use the slope formula.

$$\begin{aligned}\text{Slope} &= \frac{y_2-y_1}{x_2-x_1}\\ &= \frac{5-1}{4-2}\\ &= \frac{4}{2}\\ &= 2\end{aligned}$$

The slope of the graph is 2.



b. The line crosses the y-axis at $(0, -3)$.

The y-intercept is -3.

c. The general slope y-intercept equation is $y=mx+b$, where m is the slope and b is the y-intercept.

The slope is 2. The y-intercept is -3.

Therefore, the equation of the graph is $y=2x-3$.

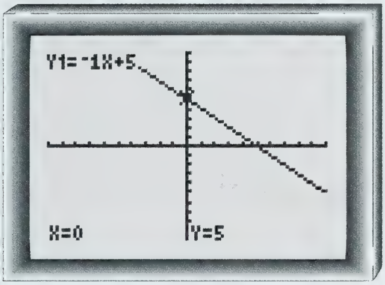
9. Textbook exercises 1.b., 1.c., 3.b., 3.e., and 3.f. of "Exercises: Checking Your Skills," p. 264

1. b. **Step 1:** Express the equation in slope y-intercept form.

$$\begin{aligned} y+x-5&=0 \\ y+x&=5 && \longleftarrow \text{Add 5 to both sides.} \\ y&=-x+5 && \longleftarrow \text{Subtract } x \text{ from both sides.} \\ y&=\underset{\substack{\uparrow\\ m}}{-1}x+\underset{\substack{\uparrow\\ b}}{5} && \longleftarrow \text{Show the coefficient of } x \text{ explicitly.} \end{aligned}$$

The slope, m, is -1 and the y-intercept, b, is 5.

Step 2: Graph the equation.

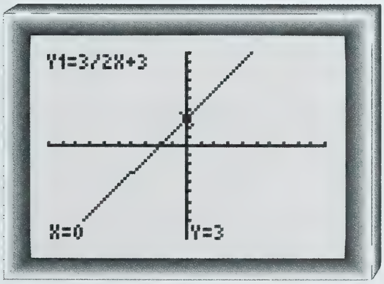


c. **Step 1:** Express the equation in slope y-intercept form.

$$\begin{aligned} 3x-2y+6&=0 \\ -2y&=-3x-6 && \longleftarrow \text{Subtract } 3x \text{ and 6 from both sides.} \\ y&=\underset{\substack{\uparrow\\ m}}{\frac{3}{2}}x+\underset{\substack{\uparrow\\ b}}{3} && \longleftarrow \text{Divide by } -2 \text{ throughout.} \end{aligned}$$

The slope, m, is $\frac{3}{2}$ and the y-intercept, b, is 3.

Step 2: Graph the equation.



Activity 1 (continued)

3. b. $-4x-2y+16=0$

$$-2y = 4x - 16 \quad \longleftarrow \text{Add } 4x \text{ and } -16 \text{ to both sides.}$$

$$y = \underset{\substack{\uparrow \\ m}}{-2}x + \underset{\substack{\uparrow \\ b}}{8} \quad \longleftarrow \text{Divide by } -2 \text{ throughout.}$$

Another way of stating the slope and *y*-intercept values is $m=-2$ and $b=8$.

The slope is -2 and the *y*-intercept is 8.

e. $3x+5y+30=0$

$$3x+5y=-30 \quad \longleftarrow \text{Subtract 30 from both sides.}$$

$$5y=-3x-30 \quad \longleftarrow \text{Subtract } 3x \text{ from both sides.}$$

$$y = \underset{\substack{\uparrow \\ m}}{-\frac{3}{5}}x \underset{\substack{\uparrow \\ b}}{-6} \quad \longleftarrow \text{Divide by 5 throughout.}$$

$m=\frac{-3}{5}$; $b=-6$

The slope is $-\frac{3}{5}$ and the *y*-intercept is -6.

f. $-2x+6y+10=0$

$$6y+10=2x \quad \longleftarrow \text{Add } 2x \text{ to both sides.}$$

$$6y=2x-10 \quad \longleftarrow \text{Subtract 10 from both sides.}$$

$$y = \underset{\substack{\uparrow \\ m}}{\frac{1}{3}}x \underset{\substack{\uparrow \\ b}}{-\frac{5}{3}} \quad \longleftarrow \text{Divide by 6 throughout and simplify.}$$

$m=\frac{1}{3}$; $b=-\frac{5}{3}$

The slope is $\frac{1}{3}$ and the *y*-intercept is $-\frac{5}{3}$.

10. a. Answers will vary. Any point you list should have an *x*-value of 4. The *y*-values can be any you choose.

The ordered pairs for three points on the line are $(4,-5)$, $(4,1)$, and $(4,5)$.

b. The equation of the graph is $x=4$.

c. The relation represented by the graph is not a function. A function may not have more than one *y*-value corresponding to any *x*-value.

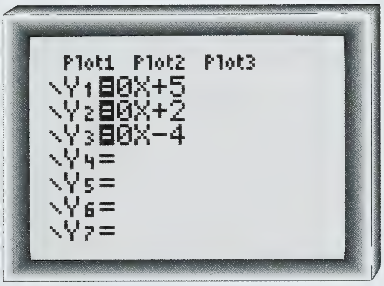
11. Textbook exercises 1 to 8 of "Investigation 3: Horizontal and Vertical Lines," pp. 261 and 262

1. Press Y= . Use the arrow keys to move to an entry.

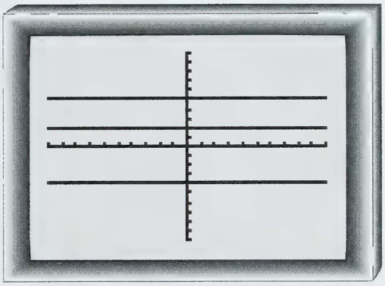
   Press CLEAR . Continue this procedure until all lines are empty.

   Return the cursor to the "Y1 = " space.

2. Enter the functions as shown and press GRAPH . The graphs look like the following.



```
Plot1 Plot2 Plot3
\Y1=0X+5
\Y2=0X+2
\Y3=0X-4
\Y4=
\Y5=
\Y6=
\Y7=
```



Use the TRACE feature to identify individual graphs. Use ▼ or ▲ (the "up" and "down" arrow keys) to move to different graphs.

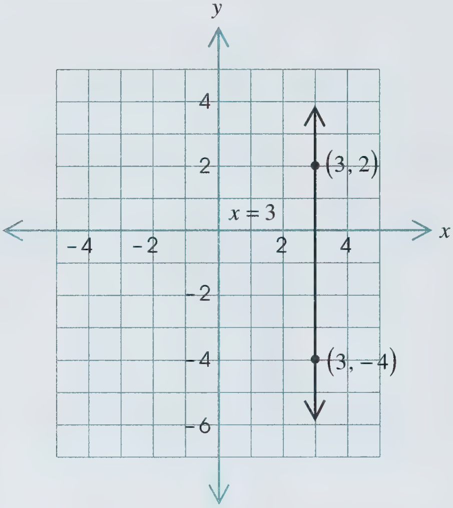
3. The slope of all the graphs is 0. The y-intercept of $y = 0x + 5$ is 5, the y-intercept of $0x + 2$ is 2, and the y-intercept of $0x - 4$ is -4.

4. As b increases, the horizontal line moves "upwards." The slope remains 0, but the y-intercept increases.

Activity 1 (continued)

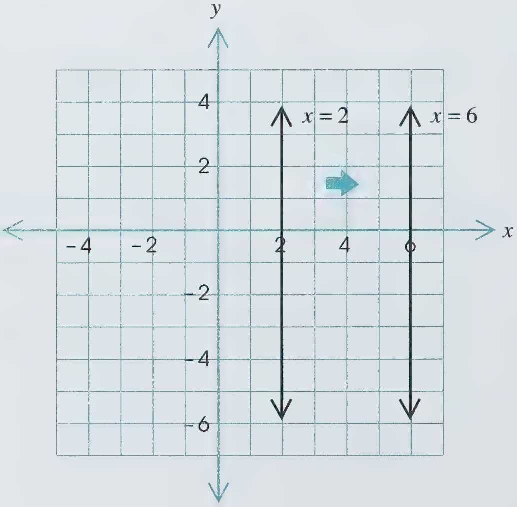
5. To graph $x = 3$, extend a (vertical) line through any two points having an x-value of 3.

The equation cannot be solved for y since y does not appear in the equation. (If y were to be included, it would have to be inserted via the term $0y$. Normally, to isolate y, you would divide throughout by 0; division by 0 is not possible.)



6. The line $x = 3$ has no defined slope. Note that "not having a slope" is different from having a slope of 0. A slope of 0 **is** defined. The graph does not cross the y-axis so the equation $x = 3$ does not have a y-intercept.

7. As a increases, the graph moves to the right.



8. For any given value of *a*, the slope of $x = a$ is not defined; the equation has no slope value. For all non-zero values of *a*, the graph of the equation does not cross the *y*-axis; the equation has no *y*-intercept.

> **Note:** When the graph of $x = a$, where $a = 0$ is drawn, the graph coincides with the *y*-axis; there is not much sense talking about its *y*-intercept.

12. Textbook exercises 3 to 6 of "Discussing the Ideas," pp. 263 and 264

3. The graph of $y = b$ is a horizontal line passing through the *y*-intercept at $(0, b)$.

4. The graph of $y = mx$ is a line passing through the origin and having a slope of *m*.

5. The equation of the vertical line is $x = a$.

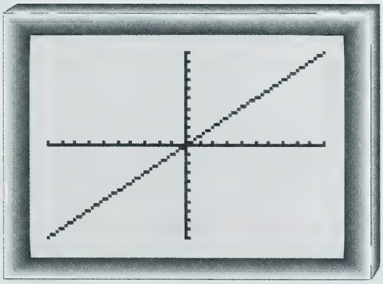
6. The equation of a straight line does not always define a function. A vertical line is **not** a function. All other lines correspond to functions.

For example, the equation $x = 6$ is not a function. Both $(6, -1)$ and $(6, 4)$ are on the line. For this relation, one input value (6) has two output values $(-1 \text{ and } 4)$. For a function, any input value can have only one output value. So $x = 6$, although linear, is not a function.

Any linear relation that can be expressed in the form $y = mx + b$ is a function.

13. Textbook exercises 2, 4, and 5 of "Exercises: Checking Your Skills." p. 264

2. This is the graph of $y = x$.



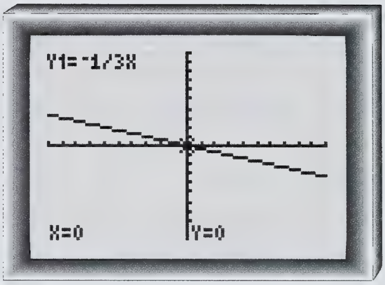
The equation $y = x$ can also be written as $y = \underset{\substack{\uparrow \\ m}}{1}x + \underset{\substack{\uparrow \\ b}}{0}$.

a. Since $m = 1$, the slope is 1.

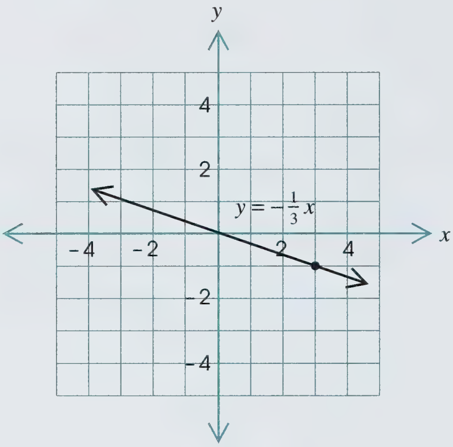
b. Since $b = 0$, the *y*-intercept is 0. The *x*-intercept is also 0.

Activity 1 (continued)

4. You could use your graphing calculator to graph the equations. Then, use the TRACE feature to locate the *x*- and *y*-intercepts, if any. Use the TRACE and TABLE features to find the domain and range of the function. If necessary, review Utility 10, "Using the TI-83 to Find the Domain and Range of a Function" on page 406 of your textbook. The calculator screen for the graph for textbook exercise 4.a. is shown to the right as an example.

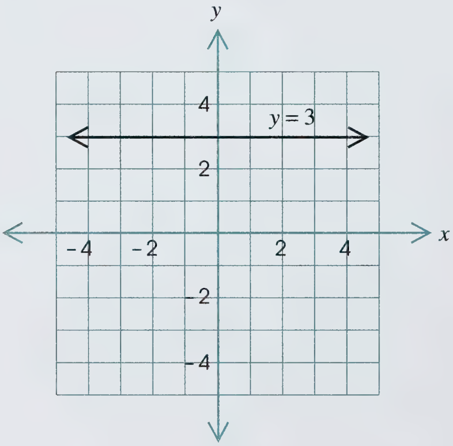


a.



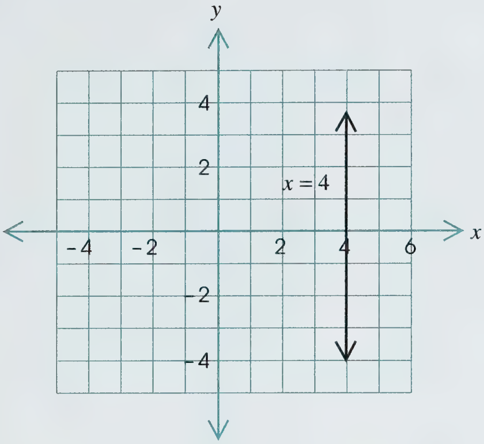
Intercepts	**Slope**
$x = 0$ $y = 0$	$-\frac{1}{3}$
Domain	**Range**
Real Numbers	Real Numbers

b.



Intercepts	**Slope**
no *x*-intercept $y = 3$	0
Domain	**Range**
Real Numbers	3

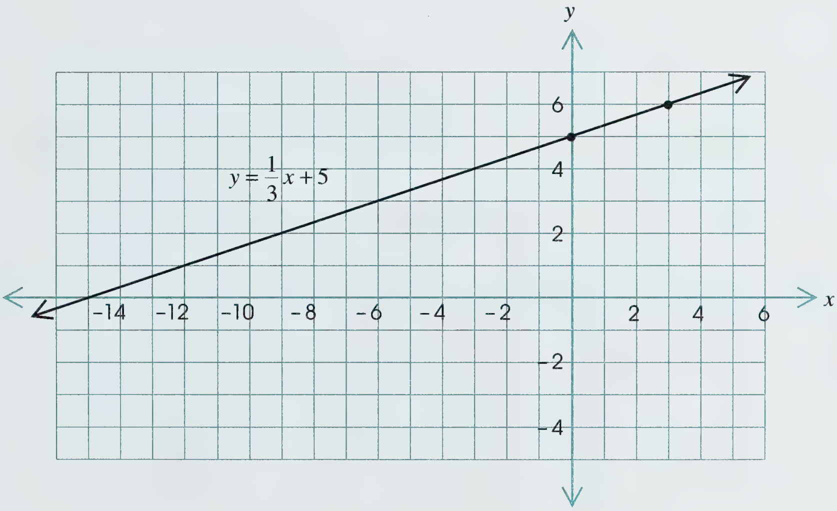
c.



Intercepts	**Slope**
$x = 4$ no y-intercept	none (undefined)

Domain	**Range**
4	Real Numbers

d.

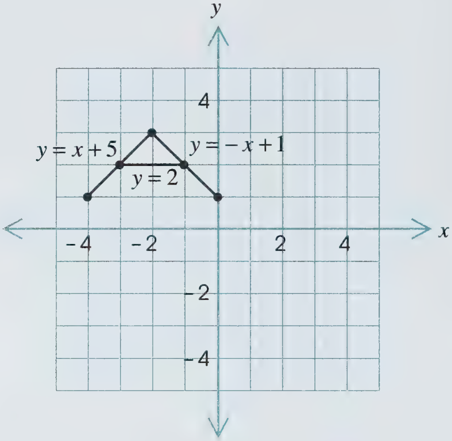


Intercepts	**Slope**
$x = -15$ $y = 5$	$\frac{1}{3}$

Domain	**Range**
Real Numbers	Real Numbers

Activity 1 (continued)

5. The graph of the equations will look as follows.



14. Based on the given information, the variables *n* and *x* can be used to label the horizontal intercept:

n-intercept ➡ *x*-intercept

15. Textbook exercise 9 of Part B of "What Should I Be Able To Do?" p. 314

9. **a.** The domain is whole numbers equal to and greater than 300.

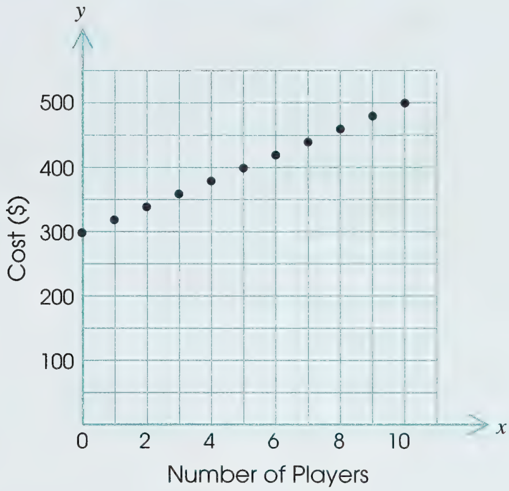
$y = \geq 300$

b. **Method 1: Drawing a Graph by Hand**

Let x be the number of players (n).
Let y be the cost (C).

x	1	2	3	4	5	6	7	8	9	10
y	320	340	360	380	400	420	440	460	480	500

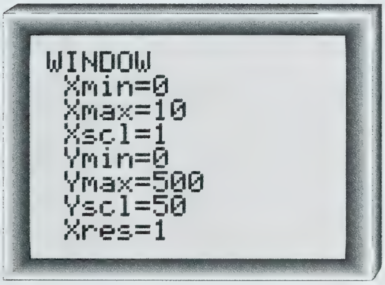
The graph of the function will look as shown.



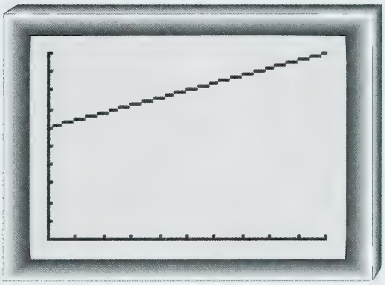
Method 2: Using a Graphing Calculator

Choose appropriate WINDOW settings and enter the equation. The given WINDOW settings display a graph as follows.

Note: The graphing calculator has been used to graph an equation; therefore, the resulting graph is a line. However, because the x-values represent players, only whole number points on the line are valid.



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=500
 Yscl=50
 Xres=1
```



c. $$\begin{aligned} \text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{500 - 300}{10 - 0} \\ &= \frac{200}{10} \\ &= 20 \end{aligned}$$

The slope is 20. The units are dollars per player.

The slope represents the extra cost for each additional player in the tournament.

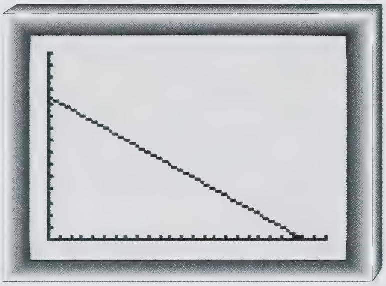
d. The *C*-intercept is another name for the *y*-intercept, or the value of *y* when $x = 0$ (or $n = 0$). The *C*-intercept is $300. This represents the fixed cost of holding the tournament. The fixed cost may include the rental of the facility, the hiring of officials, and trophies or ribbons. The fixed cost does not vary with the number of participants.

$$\begin{aligned} C &= 300 + 20n \\ &= 300 + 20(0) \\ &= 300 \end{aligned}$$

You can also use the TRACE feature on your calculator to find the *C*-intercept.

16. Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," pp. 271 and 272

1. a. The graph will look as shown at the right.



The points should be connected because neither the time (*t*) nor distance (*D*) is restricted to whole numbers.

b. The domain is all real numbers between 0 and 46 inclusive. The range is all real numbers between 0 and 23 inclusive.

c. The *D*-intercept (vertical or *y*-intercept) is 23. It represents the distance in nautical miles. The slope is -0.5. It represents the rate at which the distance between the helicopter and the boat is changing in nautical miles per minute.

Write the equation in slope *y*-intercept form.

$$\begin{aligned} y &= -0.5n + 23 \\ &= -0.5(0) + 23 \\ &= 23 \end{aligned}$$

Find the slope.

$$\begin{aligned} \text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{23 - 0}{0 - 46} \\ &= -0.5 \end{aligned}$$

You may access the TABLE view on your graphing calculator to find the coordinates of any two points. See Utility 9, "Working with Tables: Viewing a Table of Values," on page 405 of the textbook.

d. The magnitude of the slope, 0.5 nautical miles per minute, is the speed of the helicopter.

The vertical intercept, 23 nautical miles, is the distance of the helicopter from the boat at the beginning of the mission (when $t = 0$). The slope is the rate at which the distance to the boat from the helicoper is changing. The slope is negative because the distance is decreasing.

e. The horizontal intercept is the time at which the helicopter arrives at the boat. At that time, the distance to the boat, D, is 0 nautical miles. The t-intercept can be found algebraically or graphically.

Method 1: Finding the t-intercept algebraically

Let $D = 0$.

$$D = -0.5t + 23$$
$$0 = -0.5t + 23$$
$$0.5t = 23 \quad \longleftarrow \text{Add } 0.5\,t \text{ to both sides.}$$
$$t = 46 \quad \longleftarrow \text{Multiply by 2 throughout.}$$

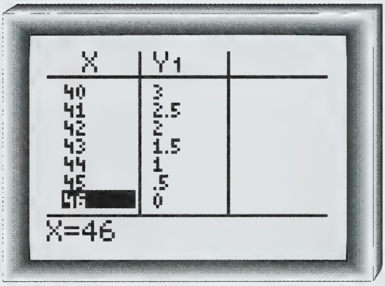
The horizontal or x-intercept is 46.

Method 2: Finding the t-intercept using the graphing calculator

Express the equation in $y = mx + b$ form.

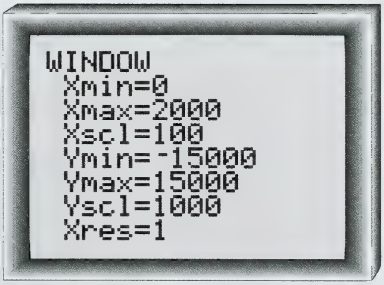
$$D = -0.5t + 23$$
$$y = -0.5x + 23$$

Enter the equation and graph the function. Then, press 2nd [TABLE] to get the table of values. Read the x-value for $y = 0$. The x-value for $y = 0$ is 46.

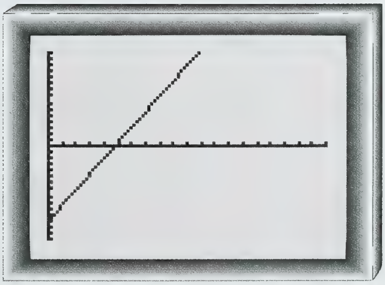


```
  X    | Y1  |
40     | 3   |
41     | 2.5 |
42     | 2   |
43     | 1.5 |
44     | 1   |
45     | .5  |
46     | 0   |
X=46
```

2. a. Following is an example of acceptable WINDOW settings. These settings will produce the given graph. The points should not be connected because the number of tickets is not continuous; this number is restricted to the counting numbers.



```
WINDOW
 Xmin=0
 Xmax=2000
 Xscl=100
 Ymin=-15000
 Ymax=15000
 Yscl=1000
 Xres=1
```



b. The price is \$25 per ticket. This is the slope of the equation.

c. The y-intercept represents the fixed costs. The promoters' fixed costs are \$12 000.

Activity 1 (continued)

d. When the promoter breaks even, the profit P equals 0.

$$\begin{aligned} P &= 25T - 12\ 000 \\ 0 &= 25T - 12\ 000 && \leftarrow \text{Subtract } 25\,T \text{ from both sides.} \\ -25T &= -12\ 000 && \leftarrow \text{Divide each side by } -25\,T. \\ T &= 480 \end{aligned}$$

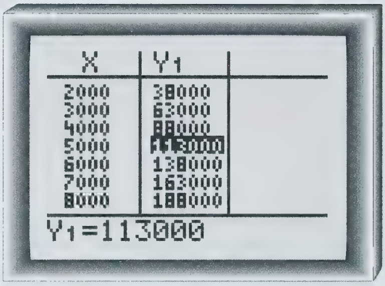
The promoter must sell 480 tickets to break even.

You may want to use the TRACE and TABLE features of your calculator to see if you can determine the break-even point and the profit when 5000 tickets are sold.

e. Find the profit algebraically or use the TABLE feature on the calculator.

$$\begin{aligned} P &= 25T - 12\ 000 \\ &= 25(5000) - 12\ 000 \\ &= 125\ 000 - 12\ 000 \\ &= 113\ 000 \end{aligned}$$

Hint: To find the value quickly, first use the TBLSET to adjust the table settings.

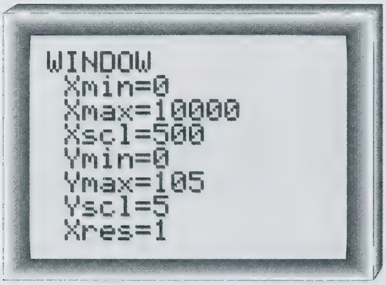


```
X      Y1
2000   38000
3000   63000
4000   88000
5000   113000
6000   138000
7000   163000
8000   188000
Y1=113000
```

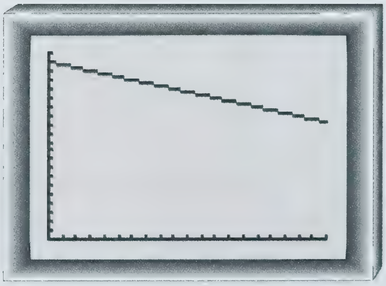
The most money the promoter could make is $113 000.

17. Textbook exercise 3 of "Exercises: Checking Your Skills," p. 272

3. **a.** A suitable WINDOW setting and the resulting graph for the equation $T = -0.0034H + 100$ are as follows.



```
WINDOW
 Xmin=0
 Xmax=10000
 Xscl=500
 Ymin=0
 Ymax=105
 Yscl=5
 Xres=1
```



Points on the graph should be connected because altitude and temperature can have fractional values.

Note: If you used a different WINDOW setting, your graph may differ. Your graph should show a vertical intercept of 100 and a downward slope. The x- and y-scales should be large enough to separate the ticks adequately for convenient reading.

b. The given formula can be rewritten as $y = -0.0034x + 100$.

The slope, -0.0034, is the coefficient of x.

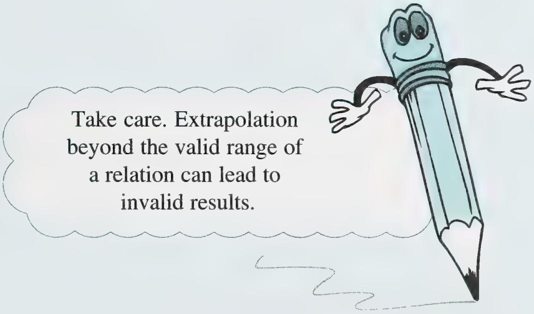
The vertical or y-intercept is 100, as indicated by the constant term in the equation.

c. The slope is expressed as degrees Celsius per metre (°C/m). The slope of -0.0034° C/m indicates a drop in the boiling point of water of 0.0034°C for every increase of 1 m of altitude.

The vertical or y-intercept, 100°C, represents the boiling point of water at sea level, when $H = 0$.

d. The horizontal intercept represents the height at which the boiling point of water is 0°C.

If you extend the graph further, you would even conclude that water would boil below its freezing temperature. The linear relation probably breaks down when you go so high that you get close to the horizontal intercept. This is where the graph gives absurd results.



Take care. Extrapolation beyond the valid range of a relation can lead to invalid results.

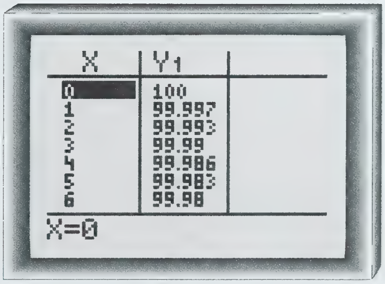
e. Follow these steps.

Step 1: Rewrite the formula in slope y-intercept form. Press [Y=] and enter the formula.

$$y = -0.0034x + 100$$

Step 2: Display the TABLE view.

Press [2nd] [TABLE].



X	Y1
0	100
1	99.997
2	99.993
3	99.99
4	99.986
5	99.983
6	99.98

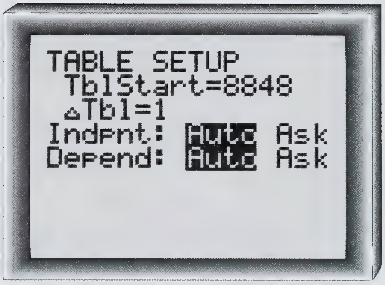
X=0

Note: Your table may start at a value other than X = 0. Your calculator will hold any previous settings.

Activity 1 (continued)

Step 3: If necessary, adjust the table to start at $X = 8848$. With the cursor in the "TblStart=" location, press the following:

2nd [TBLSET] 8 8 4 8



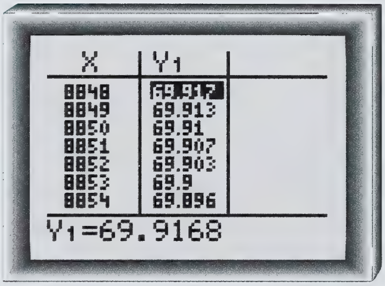
```
TABLE SETUP
 TblStart=8848
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

Note: The TABLE feature is often easier to use to find the intercepts than the TRACE feature.

Step 4: Display the TABLE view again.

Press 2nd [TABLE].

Use the arrow keys to highlight the y-value (T) when $x = 8848$ $(H = 8848)$.



```
  X     Y1
8848   69.917
8849   69.913
8850   69.91
8851   69.907
8852   69.903
8853   69.9
8854   69.896
Y1=69.9168
```

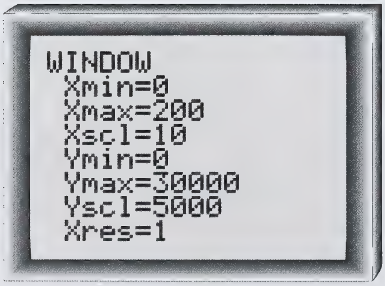
The boiling point of water on Mt. Everest is about 69.92°C.

18. **a.** Rewrite the equation in y-intercept form.

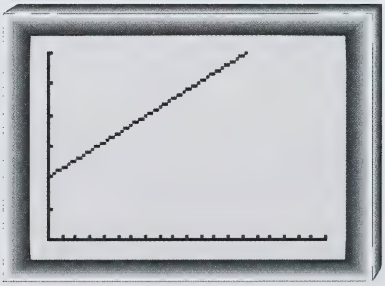
$$m = 140v + 10\ 000$$

$$y = 140x + 10\ 000$$

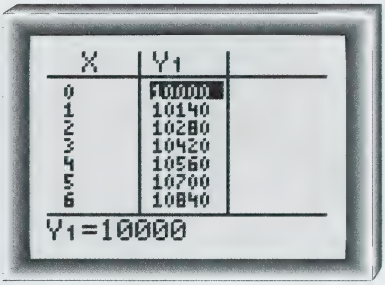
Enter the equation. Adjust the WINDOW setting as instructed. Set $\text{Xmin} = 0$ and $\text{Xmax} = 200$. Provide an appropriate Ymin and Ymax. The following is a suitable WINDOW setting and the resulting graph.



```
WINDOW
 Xmin=0
 Xmax=200
 Xscl=10
 Ymin=0
 Ymax=30000
 Yscl=5000
 Xres=1
```



b. The vertical or *y*-intercept can be found using the TABLE feature. The *y*-value for $x = 0$ is the vertical intercept.



```
X    Y1
0    10000
1    10140
2    10280
3    10420
4    10560
5    10700
6    10840
Y1=10000
```

To make the task easier, adjust the table setting to start at X=0.

Press 2nd [TBLSET].

Set the TblStart to 0.

Press 2nd [TABLE].

The vertical intercept is 10 000.

Note: This value can also be determined from the constant term of the equation or by using the TRACE feature.

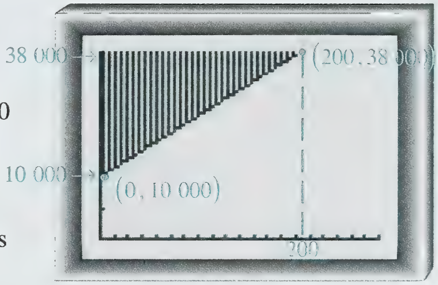
c. The vertical or *y*-intercept corresponds to the mass of the truck when its tank is empty.

d. The slope can be obtained using the TABLE feature. When *x* increases by 1, *y* increases by 140. This indicates the slope is 140. (The slope can also be determined from the coefficient of *v* in the given equation or *m* in the *y*-intercept equation. **Note:** You could also use a slope formula on your calculator.)

e. The slope represents the mass of one barrrel of oil in kilograms.

f. The range consists of values in the interval from 10 000 to 38 000 inclusive.

- If only whole numbers of barrels are carried, then these values consist of only the y_1 values for $x = 0$ to $x = 200$ in the table of part b.
- If partial barrels are allowed, then the function is continuous and the range consists of all the real numbers in the interval from 10 000 to 38 000 inclusive.



19. Question based on textbook exercise 7 of "Exercises: Checking Your Skills," p. 273

a. The vertical intercept corresponds to the fluid pressure at the surface of the water. At the water's surface, the pressure exerted is 100 kPa, which is about standard atmospheric pressure.

b. Several metres below the surface, the water pressure exerted on the chest is too high to allow unassisted breathing.

Activity 1 (continued)

20. The ratio of $\frac{\text{distance walked}}{\text{time walked}}$ is 1 to 10.

Time Walked (min)	10	25	30	60
Distance Walked (km)	1	2.5	3	6
Ratio $\frac{\text{distance}}{\text{time}}$	$\frac{1}{10}$	$\frac{2.5}{25}=\frac{1}{10}$	$\frac{3}{30}=\frac{1}{10}$	$\frac{6}{60}=\frac{1}{10}$

21. a. Pat had twice the number the cookies that Josh did. Since the number of cookies is directly related to the number of calories, Pat also had twice the number of calories.

$$2 \times 370 = 740$$

Pat consumed 740 cal.

b. The sizes of the cookies would have to be the same.

In a direct variation, the slope remains constant for a given relation. In this case, the slope corresponds to the number of calories in each additional cookie. Since the number of calories of each additional cookie is constant and the same dough was used for each, the cookie size would have to be the same.

22. Textbook exercise 5 of "Exercises: Checking Your Skills," p. 273

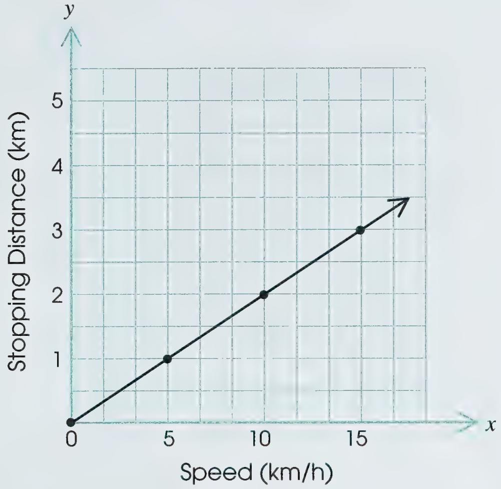
5. a. Using Pencil and Paper

If a relation is a direct variation, its graph is linear and starts at the origin.

Step 1: Create a table of values.

Speed (km/h)	0	5	10	15
Stopping Distance (km)	0	1	2	3

Step 2: Draw a graph of the relation.



Using pencil and paper requires you to calculate or estimate each value. Using a calculator simplifies this task.

Using a Graphing Calculator

Step 1: Express the relation as a function in *y*-intercept form.

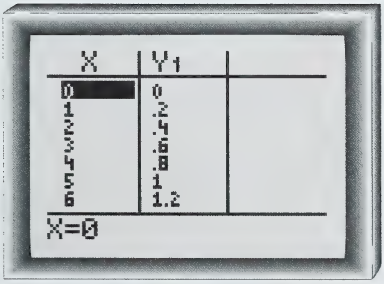
$$d = \frac{3}{15}s$$

$$= \frac{1}{5}s \quad \text{or} \quad 0.2s$$

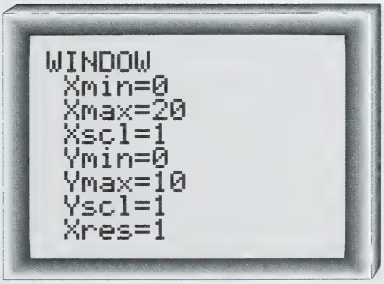
$$\therefore\ y = \frac{1}{5}x \quad \text{or} \quad 0.2x$$

Step 2: Enter the relation as $Y_1 = 0.2x$.

Step 3: Because the relation is a direct variation, it starts at the origin. Therefore, set the table of values to begin at 0. Adjust the WINDOW settings to appropriate values. One suitable WINDOW setting is shown.



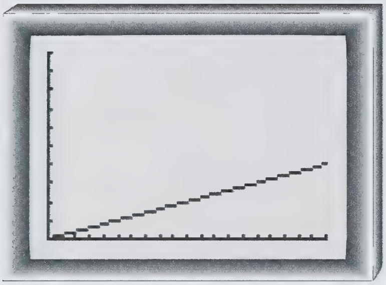
```
  X    | Y1  |
0      | 0   |
1      | .2  |
2      | .4  |
3      | .6  |
4      | .8  |
5      | 1   |
6      | 1.2 |
X=0
```



```
WINDOW
 Xmin=0
 Xmax=20
 Xscl=1
 Ymin=0
 Ymax=10
 Yscl=1
 Xres=1
```

Activity 1 (continued)

Step 4: Graph the equation.



b. Using Pencil and Paper

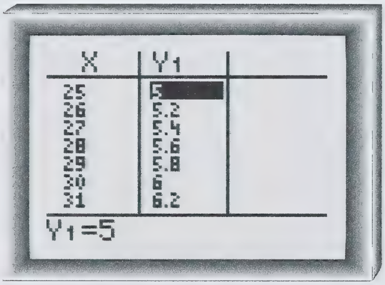
Find the value of d when s is 25.

$$\begin{aligned} d &= 0.2s \\ &= 0.2(25) \\ &= 5 \end{aligned}$$

The tanker will travel 5 km before coming to a stop.

Using a Graphing Calculator

Find the value of y when $x = 25$.



X	Y_1
25	5
26	5.2
27	5.4
28	5.6
29	5.8
30	6
31	6.2

$Y_1=5$

Adjust the TBLSET values to start at 25. This will eliminate the need to scroll.

The tanker will travel 5 km before coming to a stop.

c. Using Pencil and Paper

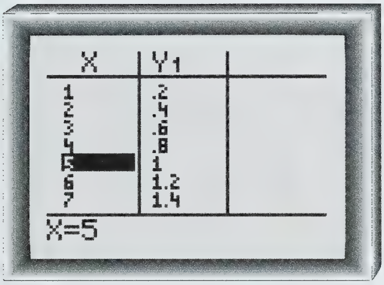
Find the value of s when d is 1.

$$\begin{aligned} d &= 0.2s \\ 1 &= 0.2s \\ s &= \frac{1}{0.2} \\ &= 5 \end{aligned}$$

The tanker is moving at 5 km/h.

Using a Graphing Calculator

Open the TABLE view and scroll to find the value of x when $y = 1$.



X	Y1
1	.2
2	.4
3	.6
4	.8
5	1
6	1.2
7	1.4

X=5

The d-value is the y-value when the formula is in the slope y-intercept form.

The tanker is moving at 5 km/h.

23. Question based on "Example 4: Direct variation," p. 270

From sales of $3000, Sam's commission is $150. Find the direct variation.

$\therefore\ C = \frac{1}{20}s$ **or** $0.05s$

Since sales and commisssions are directly related, each variable of the relation is affected by the same ratio. Find the value of C when s is 1500.

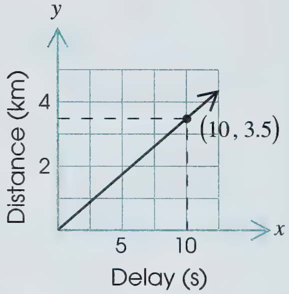
$$\begin{aligned} C &= 0.05s \\ &= 0.05(1500) \\ &= 75 \end{aligned}$$

Sam's commission on $1500 of sales is $75.

Note: You could also use the TRACE or TABLE features on your graphing calculator to find the value.

Activity 1 (continued)

24. Textbook exercise 6 of Exercises: Checking Your Skills," p. 273



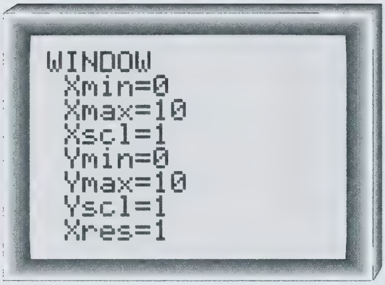
6. a. Step 1: Determine the direct variation.

$$\begin{aligned} d &= \frac{3.5}{10}t \\ &= 0.35t \end{aligned}$$

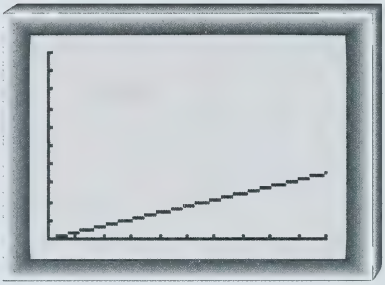
The direct variation is $d = 0.35t$.

Step 2: Rewrite the variation in y-form, set appropriate WINDOW settings, and graph the relation.

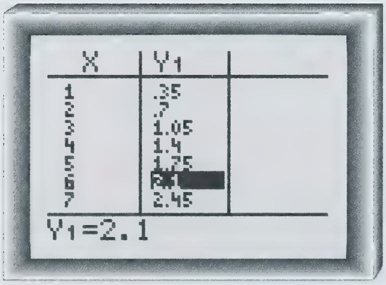
$$d = 0.35t$$
$$y = 0.35x$$



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=10
 Yscl=1
 Xres=1
```



b. Access the TABLE view and scroll down to find the value of y when $x = 6$.

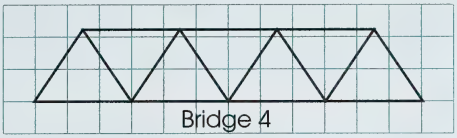


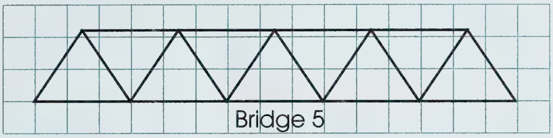
```
  X   | Y1   |
 1    | .35  |
 2    | .7   |
 3    | 1.05 |
 4    | 1.4  |
 5    | 1.75 |
 6    | 2.1  |
 7    | 2.45 |
Y1=2.1
```

Emily's friends were 2.1 km away from the storm.

25. Textbook exercises 1 to 7 of “Investigation 1: Number Patterns,” pp. 275 and 276

1. If the pattern is continued, Bridge 4 and Bridge 5 will look as follows.

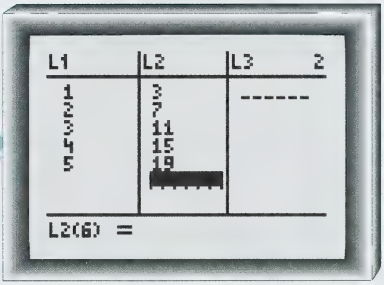




2.

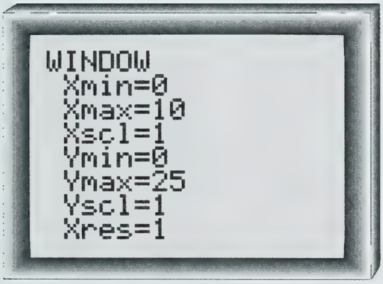
Bridge No.	No. of Struts
1	3
2	7
3	11
4	15
5	19

Enter the table columns as L1 and L2.

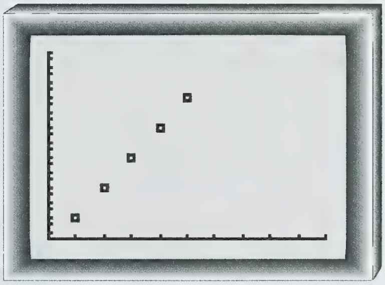


```
L1      L2      L3      2
1       3       ------
2       7
3       11
4       15
5       19
L2(6) =
```

3. Set an appropriate WINDOW setting and graph the data. An example is shown here.



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=25
 Yscl=1
 Xres=1
```



Use the TRACE feature to identify the data points.

4. The bridge number can only be natural numbers. The graph should be made up of only points above the natural numbers indicated on the horizontal axis. You cannot have a partial bridge or a partial strut; the points should not be connected.

Activity 1 (continued)

5. The entries in column 1 are consecutive natural numbers.

The next three entries in column 1 will be 6, 7, and 8.

The entries in column 2 are a sequence.

Sequence: 3 7 11 15 19

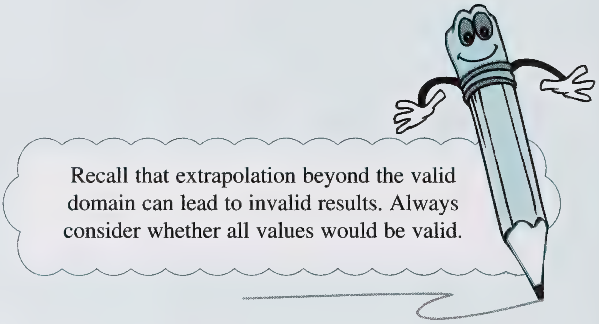
$3+4=7$ $7+4=11$ $11+4=15$ $15+4=19$

Each term in the sequence is four more than the previous term.

Therefore, the next three terms in column 2 will be as follows:

$$19+4=23$$
$$23+4=27$$
$$27+4=31$$

6. The domain corresponds to the possible bridge numbers. This consists of the natural numbers up to a certain maximum value. Beyond this value, the bridge would be too long to support itself.



Recall that extrapolation beyond the valid domain can lead to invalid results. Always consider whether all values would be valid.

7. The slope corresponds to the difference between the coordinates of any two points on the graph.

The following calculation uses the points $(4, 15)$ and $(5, 19)$:

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{19-15}{5-4}\\ &= \frac{4}{1}\\ &= 4\end{aligned}$$

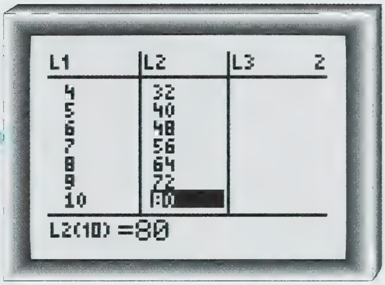
The slope is 4. This corresponds to the value added to each consecutive term.

26. Textbook exercises 1 to 8 of "Investigation 2: Photocopying Costs," p. 276

1.

Number of Copies (n)	Cost (in $) ($C$)
1	8
2	16
3	24
4	32
5	40
6	48
7	56
8	64
9	72
10	80

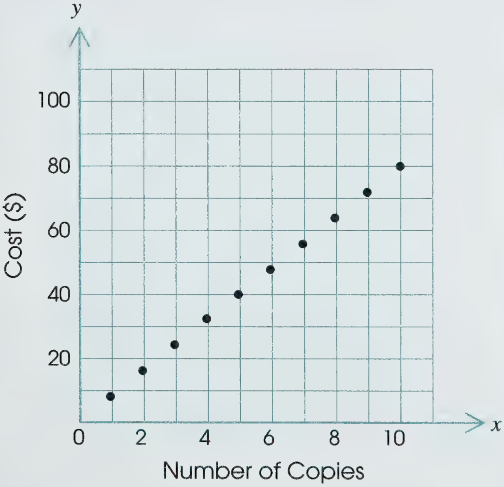
Enter the table columns as L1 and L2.

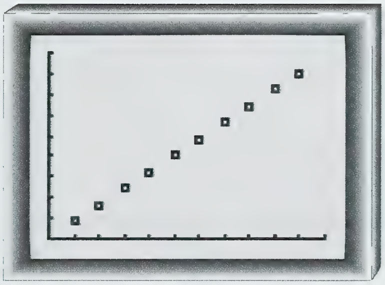


```
L1      |L2     |L3      2
  4     |  32   |
  5     |  40   |
  6     |  48   |
  7     |  56   |
  8     |  64   |
  9     |  72   |
  10    |  80   |
L2(10) =80
```

2. The domain is the natural numbers 1 to 10 inclusive.

3.





Activity 1 (continued)

4. The points should not be connected because you cannot pay a fraction of a cent or have partial copies; only whole numbers of copies are considered.

5. Use the coordinates of any two points on the graph to find the slope. The points $(2, 16)$ and $(3, 24)$ are used in the following.

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{24-16}{3-2}\\ &= \frac{8}{1}\\ &= 8\end{aligned}$$

The slope is 8.

6. The cost of *n* copies is obtained by adding 8¢ to the cost of $(n-1)$ copies.

7. The slope is numerically equal to the increase in cost referred to in textbook exercise 6.

8. The equation is $C = 8n$.

27. Answers will vary. The difference between one term and the next term of an arithmetic sequence is always the same. You can use any number to start a sequence and keep adding the same value to each term to get successive terms.

Here is one example:

$$11 \quad 19 \quad 27 \quad 35 \quad 43$$
$$+8 \quad +8 \quad +8 \quad +8$$

The slope of this linear equation is 8.

The linear function in slope *y*-intercept form is $y = 8x + b$, where *x* represents the number of the term in the sequence. When you multiply *x* by 8, you need to add 3 to get the term. Therefore, $b = 3$. The complete corresponding linear equation is $y = 8x + 3$.

28. The slope equals the difference between successive terms.

The terms 10 and 13.5 are successive. The difference between these two terms is 3.5. The difference between any other successive terms is also 3.5. Therefore, the slope of the related linear function is 3.5.

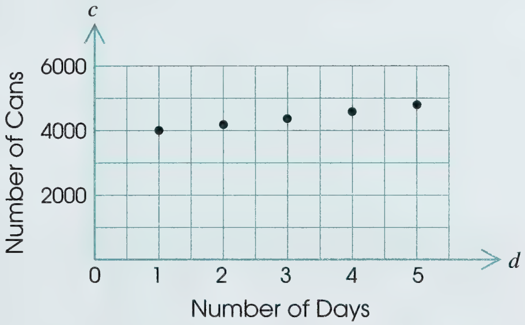
29. The slope is 12; therefore, the difference between successive terms is 12.

$$\begin{aligned} 6+12&=18\\ 18+12&=30\\ 30+12&=42 \end{aligned}$$

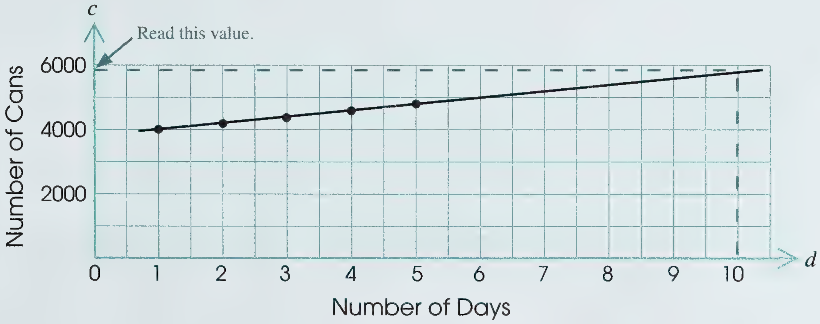
The next three terms are 18, 30, and 42.

30. Textbook exercises 1 and 3 of "Exercises: Checking Your Skills," pp. 278 and 279

1. a. Use grid paper. Draw a graph for the table with the number of days on the horizontal axis and the number of cans on the vertical axis.



b. Draw a line through the points and extend it beyond 10 days. Draw a dotted line from the point where the graph crosses the grid line for ten days back to the vertical axis. Read the number of cans from the vertical axis.



On day 10, about 5800 cans will be produced.

Activity 1 (continued)

c. Express the function as an equation and find the value when $C = 10\,000$.

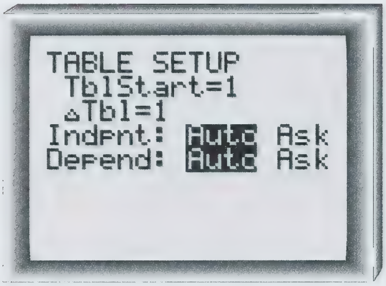
$$
\begin{aligned}
C &= 200d + 4000 \\
10\,000 &= 200d + 4000 \\
6000 &= 200d && \leftarrow \text{Subtract 4000 from each side.} \\
30 &= d && \leftarrow \text{Divide each side by 200.}
\end{aligned}
$$

The company will produce 10 000 cans on day 30.

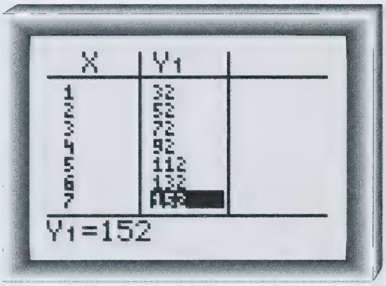
3. a. **Step 1:** Rewrite the equation in slope *y*-intercept form.

$$
\begin{aligned}
L &= 12 + 20f \\
y &= 20x + 12
\end{aligned}
$$

Step 2: Enter the equation. Set the table to begin at 1 and increase by 1. Access the TABLE view.

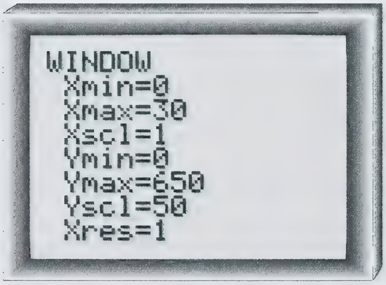


```
TABLE SETUP
 TblStart=1
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

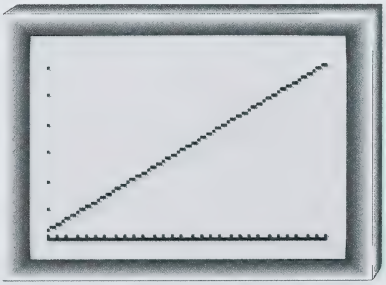


```
   X    | Y1   |
--------+------+
   1    | 32
   2    | 52
   3    | 72
   4    | 92
   5    | 112
   6    | 132
   7    | 152
Y1=152
```

b. Set appropriate WINDOW settings (the terms in L_1 represent the number of storeys; the formula is valid to a maximum of 30 storeys) and graph the equation. A suitable WINDOW setting and its corresponding graph follow. The points on the graph should not be connected. Each floor is represented by a natural number; you cannot have a partial floor.



```
WINDOW
 Xmin=0
 Xmax=30
 Xscl=1
 Ymin=0
 Ymax=650
 Yscl=50
 Xres=1
```



c. The domain of the function is the natural numbers to 30 inclusive.

d. The equation in standard form is $L = 20f + 12$.

The vertical or *y*-intercept is 12. This possibly corresponds to the length of cable involved in the driving mechanism above the top floor.

Therefore, the vertical intercept represents the cable length needed in addition to the length required according to a direct variation with the number of floors.

31. Textbook exercise "Communicating the Ideas," p. 274

All direct variations are of the form $y = mx$. This is equivalent to $y = mx + 0$, which is of the form $y = mx + b$. Therefore, all direct variations are linear functions.

For example, the relation $d = \frac{1}{3}v$, where *d* is the stopping distance and *v* is the speed, can be expressed as $y = \frac{1}{3}x$, where *y* corresponds to *d* and *x* to *v*. This relation can be expressed as $y = \frac{1}{3}x + 0$, which is a linear function.

All linear functions are not direct relations. Linear functions can be expressed as $y = mx + b$. However, for this relation to be a direct variaton, *b* must equal 0. There are many linear functions for which $b \neq 0$.

For example, the relation $C = 25n + 100$ has a vertical or *y*-intercept of 100. That is, $b = 100$. This relation is not a direct variation.

The graph of a direct variation is a line that passes through the origin $(0, 0)$.

Activity 2: Lines of Best Fit

1. **a.** The forwards farthest above the line of best fit are the following:

   Georges Laraque at $(8.3, 8)$

   Alex Selivanov at $(14.6, 32)$

   Doug Weight at $(20.3, 47)$

   b. The point farthest below the line of best fit is Rem Murray at $(13.9, 5)$.

   c. Answers will vary.

   Any two of the remaining players (from the chart) could be listed. Players whose data fall on the line of best fit would be considered most typical: Kilger, Smyth, Guerin, Grier, Moreau, and Dowd.

2. **a.** $y=-\frac{1}{2}x+4$ **b.** $y=-\frac{1}{5}x+6$ **c.** $y=x+3$

   d. $y=x+1$ **e.** $y=2x+1$ **f.** $y=\frac{1}{2}x+4$

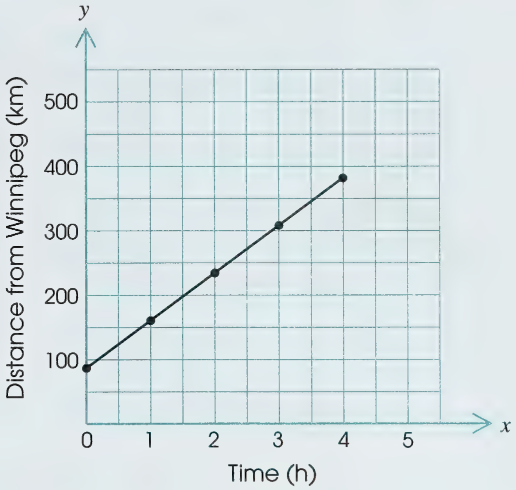
3. **Textbook exercise 1 of "Exercises: Checking Your Skills," p. 284**

   1. a.

Time (h)	Distance (km)	
0	87	
1	162	+75
2	237	+75
3	312	+75
4	387	+75
5	462	+75

Note: The motorist is driving away from Winnipeg.

b. Your graph should be similar to the one shown.



c. $$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{162 - 87}{1 - 0}\\ &= \frac{75}{1}\\ &= 75\end{aligned}$$

Tip: Often the rate in a problem turns out to be the slope of the function. Speed is a rate.

From the formula, the slope is 75. You can also conclude that the slope is 75 from the given speed.

The vertical or *y*-intercept is 87 (the value of *y* when $x = 0$).

d. The equation of the linear function will fit $y = mx + b$. The slope is 75. The vertical or *y*-intercept is 87. Therefore, the equation is $y = 75x + 87$.

4. Textbook exercises 1 and 2 of "Discussing the Ideas," p. 284

1. The slope, by definition, is rise over run. In the function of Example 2, the rise, or change of the vertical component, is the change in distance. The run, or change of the horizontal component, is the change in time. However, $\frac{\text{change in distance}}{\text{change in time}}$ is, by definition, the speed. Therefore, in Example 2, the slope equals the speed. (See the Tip given in question 3.c.)

The car travelled 180 km in 2 h. Therefore, the car started its journey 50 km $(230\text{ km} - 180\text{ km})$ from Winnipeg. You could also use the *y*-intercept on the graph to get this value.

2. Any point on the line satisfies the equation for the line.

Activity 2: (continued)

5. **Textbook exercises 2, 3.a., and 3.c. of "Exercises: Checking Your Skills," pp. 284 and 285**

   2. Let *p* be the number of pages still to be read and *t* be the time in minutes spent reading.

   The rate of reading is 0.75 pages/min. So, the rate by which the unread pages diminishes is -0.75 pages per minute. This negative rate determines the slope. (The negative slope of the graph confirms the negative sign of the slope of the equation.)

   The relation of the number of unread pages as a function of time can be expressed as follows:

   $p=-0.75t+b$ ← The value of b is still to be determined.

   Solve for *b*. Choose a point on the line, for example, $(10, 35)$. When $t=10$, then $p=35$.

   $p=-0.75t+b$

   $35=(-0.75)(10)+b$ ← Substitute the values of the coordinates in point (10, 35) into the equation.

   $35=-7.5+b$

   $b=42.5$

   Therefore, the complete equation for the relation is $p=-0.75t+42.5$.

   3. **a.** After 1 h, the driver is 120 km from Winnipeg.

   He travelled 100 km in the 1 h.

   $120-100=20$

   One hour earlier, the driver was 20 km from Winnipeg.

   c. Let *d* be the distance in kilometres from Winnipeg and *t* be the time travelled in hours. The slope is given by the speed. The speed is 100 km/h.

   When $d=120$, then $t=1$.

   $d=100t+b$

   $120=100(1)+b$ ← Substitute the known quantities.

   $b=20$ ← Solve for b.

   Therefore, $d=100t+20$ is the equation of the function.

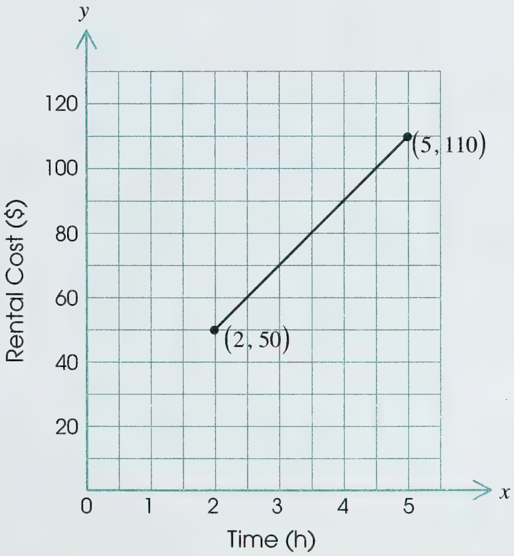
6.
$$\begin{aligned} y &= 3x+9 \\ &= 3(4)+9 && \longleftarrow \text{Substitute for } x \text{ in the linear equation.} \\ &= 12+9 \\ &= 21 && \longleftarrow \text{This is the second coordinate of one of the points in the preceding example.} \end{aligned}$$

7.
$$\begin{aligned} s &= 3h-22 \\ 22 &= 3h-22 && \longleftarrow \text{Substitute for } s. \\ 44 &= 3h && \longleftarrow \text{Add 22 to both sides.} \\ 3h &= 44 && \longleftarrow \text{Reverse the equation.} \\ h &= \frac{44}{3} && \longleftarrow \text{Divide both sides by 3.} \\ &= 14\frac{2}{3} \end{aligned}$$

Shaquille O'Neal's foot is $14\frac{2}{3}$ inches long.

8. Textbook exercises 4 and 5 of "Exercises: Checking Your Skills," p. 285

4. **a.** The graph will look as follows.



Activity 2 (continued)

b. The hourly rate corresponds to the slope.

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{110-50}{5-2}\\ &= \frac{60}{3}\\ &= 20\end{aligned}$$

Therefore, the hourly rate is \$20/h.

The fixed premium is the vertical or *y*-intercept. To find the *y*-intercept, *b*, express the relation as an equation in $y = mx + b$ form and solve for *b*.

Let *C* be the cost of the rental in dollars and *t* be the time in hours.

$$\begin{aligned}C &= 20t + b\\ 50 &= 20(2) + b && \longleftarrow \text{Substitute with the coordinates of one of the points.}\\ 50 &= 40 + b && \longleftarrow \text{Simplify.}\\ b &= 10 && \longleftarrow \text{Isolate } b.\end{aligned}$$

The fixed premium is \$10.

5. **a.** If simple interest, *y*, varies directly as the amount borrowed, *x*, then $y = mx$.

The amount \$325 is 3.25 times as much as \$100.

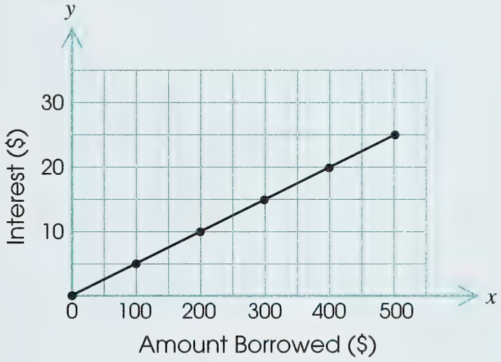
The interest will increase by the same factor, since the relation is a direct variation. Therefore, the interest on \$325 is $\$5 \times 3.25 = \$16.25$.

b. A possible table of values follows. You may have chosen different values for your table.

	Amount Borrowed ($)	Interest ($)	
	0	0	
	100	5	
100 × 2 →	200	10	← 5 × 2
100 × 3 →	300	15	← 5 × 3
100 × 4 →	400	20	← 5 × 4
100 × 5 →	500	25	← 5 × 5

c. The graph of the relation will look as shown.

Recall, the graph of a direct variation goes through the origin.



An equation of direct variation is of the form $y = mx$. Let y be the interest and x the amount borrowed. To determine m, substitute for x and y according to the coordinates of a known point other than $(0, 0)$.

Substitute the coordinates of point $(100, 5)$ into the equation.

$$\begin{aligned} y &= mx \\ (5) &= m(100) && \longleftarrow \text{Substitute values for } x \text{ and } y. \\ m &= \frac{5}{100} && \longleftarrow \text{Solve for } m. \\ &= 0.05 \end{aligned}$$

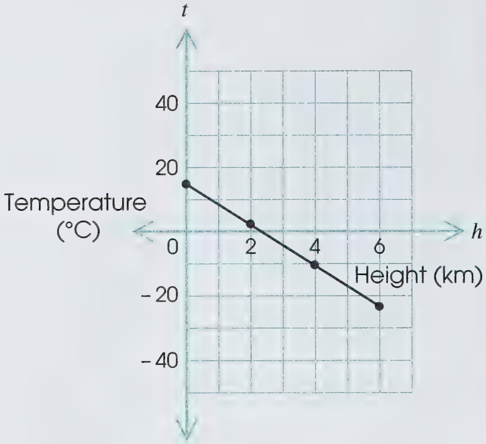
Remember, $b = 0$ for direct variations.

The equation for the relation is $y = 0.05x$.

Activity 2 (continued)

9. Textbook exercises 10.a. to 10.d. of "Exercises: Extending Your Thinking," p. 287

10. a. A table of values and the corresponding graph will look as follows.



Height (km)	Temperature (°C)	
0	15	+6.5
1	8.5	+6.5
2	2	+6.5
3	−4.5	+6.5
4	−11	+6.5
5	−17.5	+6.5
6	−24	

Temperature increases going down towards sea level.

Let *t* be the temperature in °C and *h* be the height in kilometres.

Then $t = mh + b$, where *m* and *b* are still to be determined.

The slope is -6.5, because the rate is -6.5 °C/km in altitude.

Use a point on the graph to determine *b*. When $h = 6$, then $t = -24$.

$$t = mh + b$$
$$(-24) = -6.5(6) + b \quad \leftarrow \text{Substitute known values.}$$
$$-24 = -39 + b \quad \leftarrow \text{Simplify.}$$
$$15 = b \quad \leftarrow \text{Add 39 to both sides.}$$
$$b = 15 \quad \leftarrow \text{Reverse sides.}$$

The equation of the relation is $t = -6.5h + 15$.

b. At sea level, $h = 0$. Substitute this value in the equation.

$$t = -6.5h + 15$$
$$= -6.5(0) + 15$$
$$= 15$$

At sea level, the temperature on April 24 was 15°C.

c. Convert the height to kilometres. The height in the formula is given in kilometres.

$$1000 \text{ m} = 1 \text{ km}$$
$$5951 \text{ m} = 5.951 \text{ km}$$

$$\begin{aligned} t &= -6.5h + 15 \\ &= -6.5(5.951) + 15 && \longleftarrow \text{Substitute the value for } h. \\ &\doteq -23.68 && \longleftarrow \text{Solve for } t. \end{aligned}$$

The temperature at the summit at Mt. Logan was about $-23.7°\text{C}$.

d. Using the graph, you can estimate the temperature was 0°C at an elevation of around 2 km.

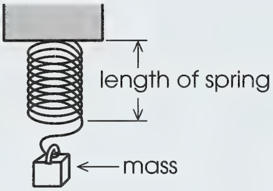
A more precise answer can be obtained using the equation.

$$\begin{aligned} t &= -6.5 + 15 \\ 0 &= -6.5h + 15 && \longleftarrow \text{Substitute the value for } t. \\ 6.5h &= 15 && \longleftarrow \text{Add 6.5 } h \text{ to both sides.} \\ h &= \frac{15}{6.5} && \longleftarrow \text{Divide both sides by 6.5.} \\ &\doteq 2.31 \end{aligned}$$

The altitude at which the temperature was 0°C on April 24 is about 2.3 km.

10. Textbook exercises 1 to 6 of "Investigation 1: The Spring," p. 288

Your answers will vary depending on the data you collected. The stiffness of the spring or elastic band and its normal length will affect results.



Sample data is given here.

1. The spring used for the sample data is 15 cm in length.

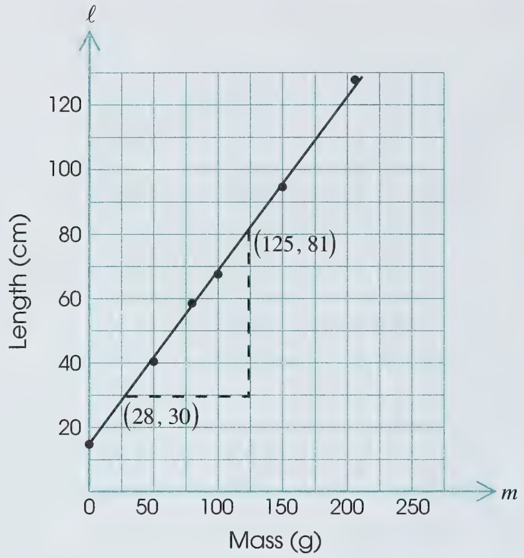
2. and 3. A 50-g mass was attached to the spring. The length of the spring was 40 cm.

The data collected was put into a table.

Mass (g)	0	50	78	100	150	208
Length (cm)	15	40	59	69	95	127

Activity 2 (continued)

4. and 5. The scatterplot and line of best fit based on the sample data are shown in the following.



Choose two points on the line of best fit.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &\doteq \frac{81 - 30}{125 - 28} \\ &\doteq \frac{51}{97} \\ &\doteq 0.53 \end{aligned}$$

The slope is about 0.53.

The vertical or *y*-intercept is 15, according to where the line of best fit meets the vertical axis. This value represents the length of the unloaded spring.

6. Answers may vary. Let ℓ be the length of the spring in centimetres. Let *m* be the mass in grams of the attached object. Then, the relation can be expressed as $\ell = 0.53\,m + 15$.

11. Textbook exercises 1 to 6 of "Investigation 2: The Candle," p. 289

Answers will vary. The original length of your candle, its circumference, and its composition will affect your results. A small, slender candle works best for this investigation. Using a very "fat" candle (one with a large diameter) may require readings taken over a much longer time.

One example follows.

1. The candle used in the sample was 8 cm in length.

2. Data collected in this sample follows.

Time (min)	Length (cm)
0	8.0
2	6.6
4	5.5
6	4.1
8	3.2
10	2.2

You may wish to set up a suitable table before lighting the candle.

3. and 4. The scatterplot and estimated line of best fit for the sample follow.

Find the slope.

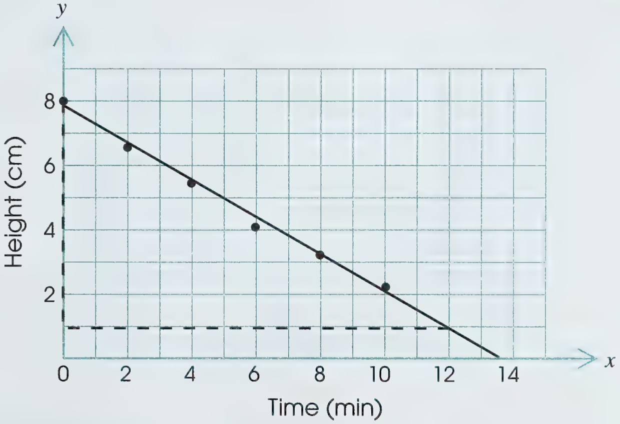
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - 7.9}{12 - 0}$$

$$\doteq \frac{-6.9}{12}$$

$$\doteq -0.6$$

The slope is about -0.6.



For greatest accuracy, choose the largest possible triangle that lies on grid lines.

Activity 2 (continued)

5. From the estimated line of best fit, the vertical or y-intercept is 7.9. That is where the line meets the vertical axis.

   The general form of the linear equation is $y = mx + b$. Let h be the height (length) of the candle in centimetres and t be the time in minutes that the candle burns.

$$\begin{aligned} y &= mx + b \\ h &= mt + b && \longleftarrow \text{Substitute with appropriate variables.} \\ h &= -0.6t + 7.9 && \longleftarrow \text{Substitute specific values for } m \text{ and } b. \end{aligned}$$

   The equation for this line of best fit is $h = -0.6t + 7.9$.

6. When the candle burns down to the plasticine, $h = 0$.

$$\begin{aligned} h &= -0.6t + 7.9 \\ 0 &\doteq -0.6t + 7.9 && \longleftarrow \text{Substitute for the specific value of } h. \\ 0.6t &\doteq 7.9 && \longleftarrow \text{Add } 0.6\,t \text{ to both sides.} \\ t &\doteq \frac{7.9}{0.6} && \longleftarrow \text{Divide both sides by } 0.6. \\ &\doteq 13 && \longleftarrow \text{Simplify.} \end{aligned}$$

   Based on the sample data, the candle will burn down to the plasticine in about 13 min.

12. Textbook exercises 1, 3, and 4 of “Discussing the Ideas,” p. 290

1. Answers will vary; the proximity of the points to the line will depend on your experimental results.

   - Data points may vary from the trend line because of measuring error.
   - In the spring investigation, once the spring is significantly stretched, further stretching may become more difficult. This could cause data points on the right side of the graph to be below the line of best fit.
   - A variation in the circumference of the candle or the buildup of wax could affect the rate at which the candle shortens. This could cause the data to deviate from the line of best fit without an apparent pattern.

3. The slopes represent rates of change. In Investigation 1, the slope represents the stretch of the spring with respect to an increase of mass suspended. In Investigation 2, the slope represents the rate at which the candle shortens.

4. Assuming no other differences (for example, a softer wax), if a candle of bigger diameter were used, the candle would shorten at a slower rate. Such a rate would be reflected by a slope having smaller magnitude, that is, a slope closer to 0. The line graph would be less steep. There is a greater likelihood that a candle with a larger diameter will develop a buildup of wax before shortening—this could skew data points.

   Waxes burn at different rates. The line graph for a slow-burning wax would incline downwards much less than the line graph for a fast-burning wax.

13. a. Step 1: Arrange the data from least to greatest.

2, 3, 3, 5, 9, 11, 15, 21, 22

Step 2: There is an odd number of values; circle the middle value.

2, 3, 3, 5, ⑨, 11, 15, 21, 22

The median is 9.

b. Step 1: Arrange the data from least to greatest.

14, 20, 21, 30, 31

Step 2: There is an odd number of values; circle the middle value.

14, 20, ㉑, 30, 31

The median is 21.

c. Step 1: Arrange the data from least to greatest.

0, 1, 1, 3, 3, 3, 6, 7, 8, 8, 9, 10

Step 2: There is an even number of values; place a line segment between the middle values.

0, 1, 1, 3, 3, 3,|6, 7, 8, 8, 9, 10

Step 3: Find the mean of the two values next to the line segment.

$$\begin{aligned}\frac{3+6}{2} &= \frac{9}{2}\\ &= 4.5\end{aligned}$$

The median is 4.5.

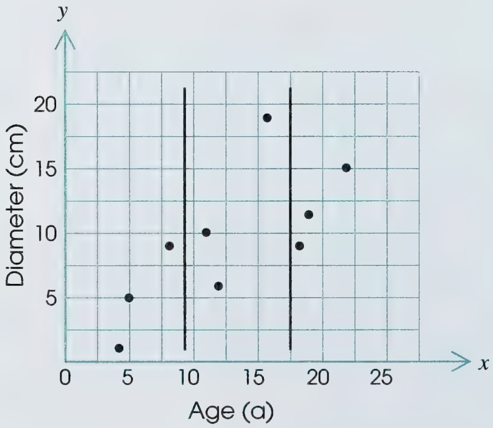
Activity 2 (continued)

14. Textbook exercises 1 to 9 of "Investigation: The Median-Median Line of Best Fit," pp. 294 to 296

1. Following are the data points arranged in ascending order, based on the values of the x-coordinate.

Age x (years)	Diameter y (cm)
4	1
5	5
8	9
11	10
12	6
16	19
18	9
19	12
22	15

2. and 3. Your graph and the line segments dividing the data points into equal groups should look like the following.



4. **Group 1 Points:** $(4, 1), (5, 5), (8, 9)$

Arrange the x-values of the Group 1 points in ascending order. Find the median of the x-values. There is an odd number of values. Circle the middle value.

4, ⑤, 8

The median of the x-values is 5.

Arrange the y-values of the Group 1 points in ascending order. Find the median of the y-values. There is an odd number of values. Circle the middle value.

1, ⑤, 9

The median of the y-values is 5.

Therefore, the summary point of Group 1 is $(5, 5)$.

Group 2 Points: $(11, 10), (12, 6), (16, 19)$

Find the median of the x-values. The median of the x-values is 12.

11, ⑫, 16

Find the median of the y-values. The median of the y-values is 10.

6, ⑩, 19

Remember to place the values in order from least to greatest.

The summary point of Group 2 is $(12, 10)$.

Group 3 Points: $(18, 9), (19, 12), (22, 15)$

Find the median of the x-values. The median of the x-values is 19.

18, ⑲, 22

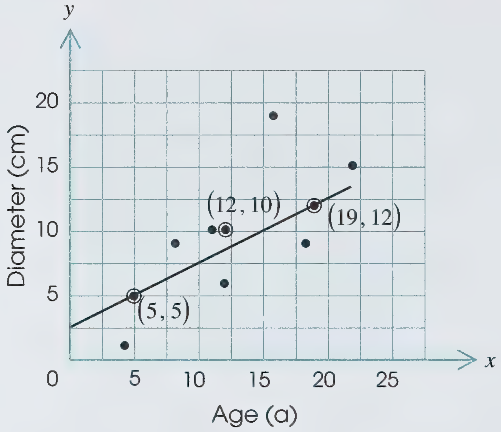
Find the median of the y-values. The median of the y-values is 12.

9, ⑫, 15

The summary point of Group 3 is $(19, 12)$.

Activity 2 (continued)

5. Your labelled graph and the line passing through the outer two summary points will look like the example shown.



Note: Extend the line to the *y*-axis in order to determine the *y*-intercept.

6. Determine the *y*-intercept from the graph. The *y*-intercept is about 2.5.

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{12-5}{19-5}\\ &= \frac{7}{14}\\ &= 0.5\end{aligned}$$

The slope is 0.5.

Therefore, the equation of the line is $y = 0.5x + 2.5$. (You could also express the equation in fraction form: $y = \frac{1}{2}x + 2\frac{1}{2}$.)

7. The summary point of Group 2 is $(12, 10)$. Find the y-coordinate of the point directly below this summary point and on the line drawn between the endpoints.

$y = 0.5x + 2.5$
$\quad = 0.5(12) + 2.5$ ← Substitute the x-value of the point into the equation of the line.
$\quad = 6 + 2.5$
$\quad = 8.5$

The y-coordinate of this point is 8.5.

8. Find the vertical distance between the Group 2 summary point and the point on the line directly below this point. The summary point is $(12, 10)$; the line point is $(12, 8.5)$.

$10 - 8.5 = 1.5$

The vertical distance is 1.5.

The y-coordinate of a point $\frac{1}{3}$ of the distance to the summary point would be at

$8.5 + \frac{1}{3}(1.5) = 8.5 + 0.5 = 9$.

Therefore, the coordinates of the point would be $(12, 9)$.

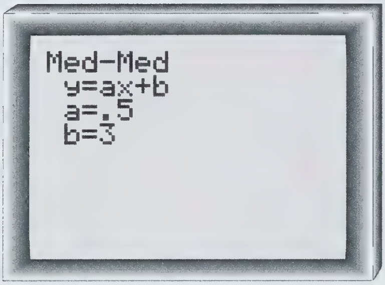
9. The equation of the line through the outer midpoints is $y = 0.5x + 2.5$. The parallel line shifted towards the middle summary point is $y = 0.5x + b$, for some value of b. The new line passes through $(12, 9)$.

$9 = 0.5(12) + b$ ← Substitute specific values of x and y based the coordinates of the point to be on the line.
$9 = 6 + b$ ← Add -6 to both sides.
$b = 3$ ← Simplify.

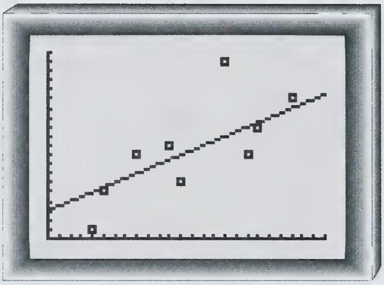
The equation of the line closer to the middle summary point is $y = 0.5x + 3$.

15. Textbook exercise 3 of "Discussing the Ideas," p. 297

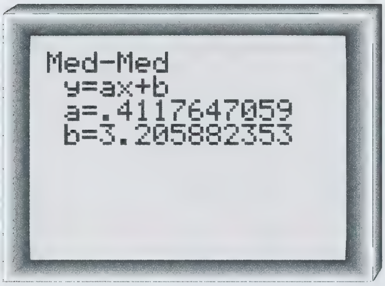
3. The original equation and graph were as follows.



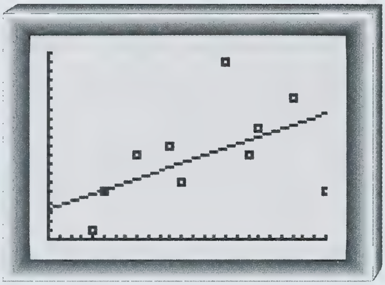
```
Med-Med
 y=ax+b
 a=.5
 b=3
```



Compare these with the "outlier" equation and graph.



```
Med-Med
 y=ax+b
 a=.4117647059
 b=3.205882353
```



The right portion of the line on the screen dropped a small amount and the slope is a little less steep than before the addition of the outlier.

Look at the third last point on the right. Think of it as a reference point. After the outlier is included, the line of best fit is slightly below this reference point. Before the outlier was added, the line lay just above this reference point.

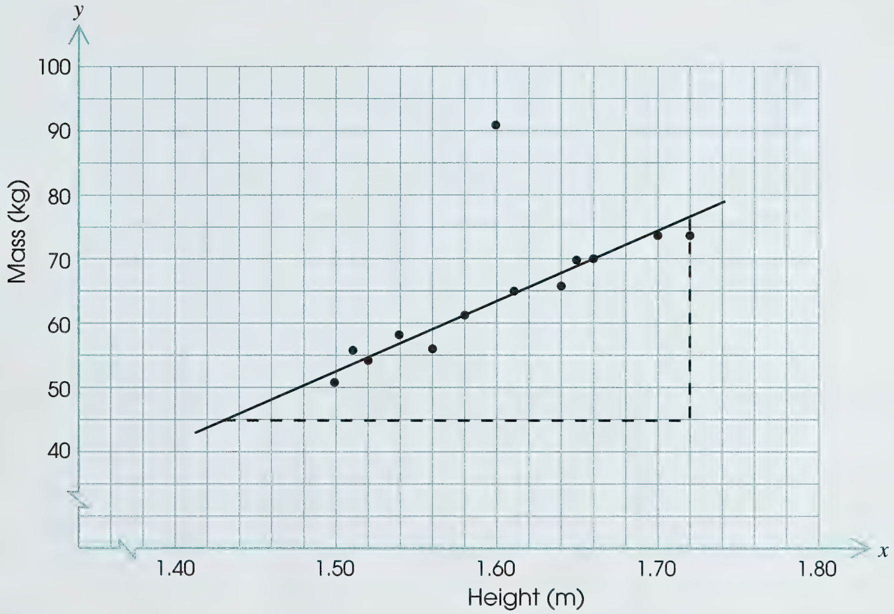
The outlier "pulled" the line down a small amount.

The original equation of the line was $y = 0.5x + 3$; the equation of the outlier line is $y = 0.411\,764\,705\,9x + 3.205\,882\,353$ or about $y = 0.4x + 3.2$.

16. Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," pp. 297 and 298

1. a. Using estimation to find the line of best fit

Plot the data. Draw a line of best fit placed so that there are close to the same number of points above and below it. Your estimated line may be somewhat different from the one shown. **Note:** A zig-zag mark is used to indicate the break in the scale.



Find the slope by choosing two points that are on the estimated line of best fit, for example, $(1.72, 77)$ and $(1.43, 45)$.

$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1}\\ &\doteq \frac{77 - 45}{1.72 - 1.43}\\ &\doteq \frac{32}{0.29}\\ &\doteq 110\end{aligned}$$

The slope is about 110.

To find the value of b, use one of the points on the line and the equation of a line, $y = mx = b$. The line of best fit is not extended to the y-axis to read the slope since the values on the axes do not start at the origin.

Using the point $(1.72, 77)$, the value of b is calculated as follows:

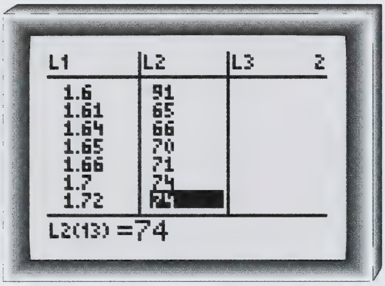
$$\begin{aligned} y &= mx + b \\ 77 &\doteq 110(1.72) + b \\ 77 &\doteq 189.2 + b \\ b &\doteq 77 - 189.2 \\ &\doteq -112 \end{aligned}$$

Use the unrounded value of the slope in your calculations.

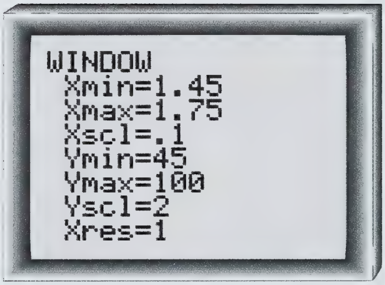
Therefore, the equation for the estimated line of best fit is $y = 110x - 112$.

b. Using the median-median method to find the line of best fit

Step 1: Access the list editor and enter the data. Enter height values as L1 data and mass values as L2 data. Adjust the WINDOW settings to appropriate values. One example is shown.

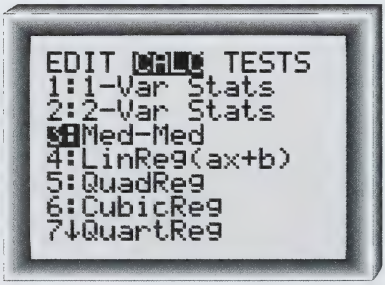


```
L1     |L2    |L3      2
 1.6   | 91   |
 1.61  | 65   |
 1.64  | 66   |
 1.65  | 70   |
 1.66  | 71   |
 1.7   | 74   |
 1.72  | 74   |
L2(13) =74
```

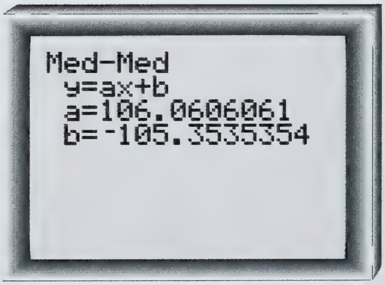


```
WINDOW
 Xmin=1.45
 Xmax=1.75
 Xscl=.1
 Ymin=45
 Ymax=100
 Yscl=2
 Xres=1
```

Step 2: Press STAT ▶ (the right arrow key). Press 3 to select the median-median line of best fit. Press ENTER to display the equation of the line.



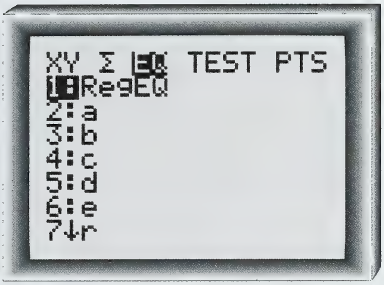
```
EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
```

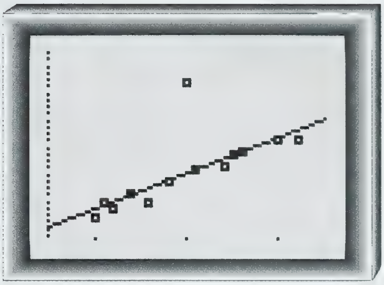


```
Med-Med
 y=ax+b
 a=106.0606061
 b=-105.3535354
```

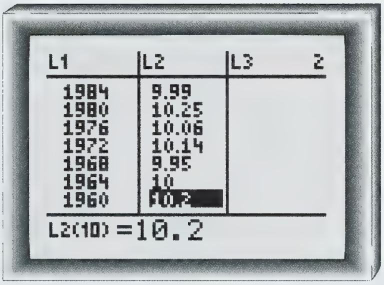
The equation of the line of best fit is about $y = 106x - 105$. The values of m and b in this equation are reasonably close to the values of m and b in the equation obtained from the estimated line of best fit in part 1.a.

Step 3: Graph the scatterplot and line of best fit. Press Y= VARS 5. Use the right arrow key to highlight EQ. Press ENTER and GRAPH.



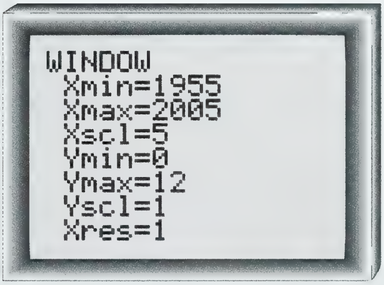


2. a. **Step 1:** Access the list editor. Clear any data in the lists. Enter the year of each record in L1 and the winning times in L2. Adjust the WINDOW settings to appropriate values. One example is shown.



L1	L2	L3 2
1984	9.99	
1980	10.25	
1976	10.06	
1972	10.14	
1968	9.95	
1964	10	
1960	10.2	

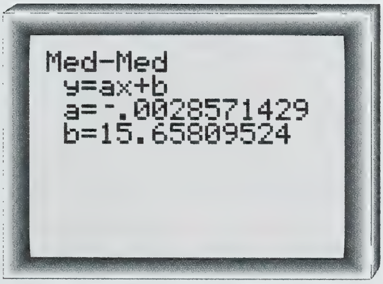
L2(10) =10.2



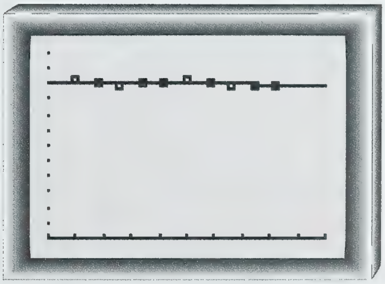
```
WINDOW
 Xmin=1955
 Xmax=2005
 Xscl=5
 Ymin=0
 Ymax=12
 Yscl=1
 Xres=1
```

Step 2: Determine the coefficients for the median-median line of best fit.

Press STAT ▶ 3 ENTER. The equation for the median-median line of best fit is about $y = -0.003x + 15.7$. Graph the scatterplot and the line of best fit on the same screen. If necessary, refer to the instructions in Step 2 and Step 3 of exercise 1.b.



```
Med-Med
 y=ax+b
 a=-.0028571429
 b=15.65809524
```

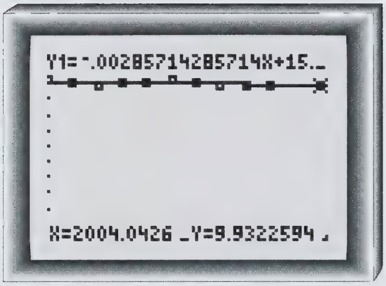


Activity 2 (continued)

Step 3: Make a prediction for the winning time in 2004.

Method 1: Using the TRACE feature

Press TRACE. Use the arrow keys to find a point along the line of best fit where x is about 2004. (Press ▲ to move the tracer to the line; then, press ▶ to move the tracer to the right.)

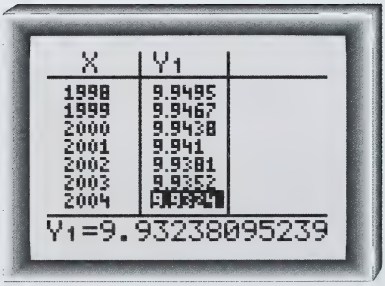


```
Y1=-.0028571428571 4X+15._
X=2004.0426 _Y=9.9322594
```

The winning time for 2004 is about 9.93 s.

Method 2: Using the TABLE feature

Press 2nd [TBLSET] to set the table start close to or on 2004. Then, press 2nd [TABLE] to view the table. Find the value of y when $x = 2004$.



```
  X   | Y1     |
1998  | 9.9495 |
1999  | 9.9467 |
2000  | 9.9438 |
2001  | 9.941  |
2002  | 9.9381 |
2003  | 9.9352 |
2004  | 9.9324 |
Y1=9.93238095239
```

The winning time for 2004 is about 9.93 s.

b. The median-median line of best fit is a good representation of the relation if it is linear. Note that the slope of the median-median line of best fit has a slope of less than zero.This means the line will eventually cross the x-axis. For that year, the winning time would be 0. That value is absurd.

At most, you could say the relation between winning time and year is linear for a limited span of time.

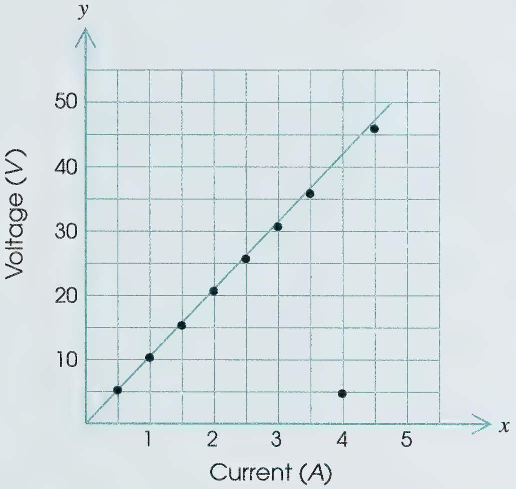
17. **a.** The point $(63, 29)$ has the largest residual. The residual of point $(63, 29)$ is about 8.

b. If the line were to be moved up towards $(63, 29)$, the squares of residuals of points below the line—$(67, 17)$ and $(77, 22)$—would increase; those of points above the line—$(63, 29)$, $(70, 26)$, $(72, 30)$, $(74, 30)$, and $(81, 32)$—would decrease.

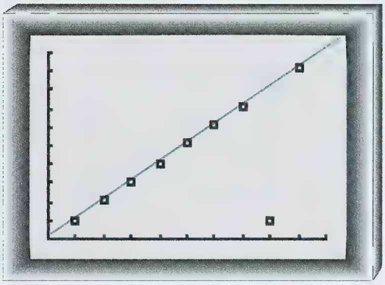
18. **Textbook exercises 1 to 5 of "Investigation 2: Compare the Lines of Best Fit Obtained Using Different Methods," p. 302**

1. and 2. Answers may vary slightly. The scatterplot and line of best fit should be similar to the following.

Using a Coordinate Grid



Using a Graphing Calculator



Activity 2 (continued)

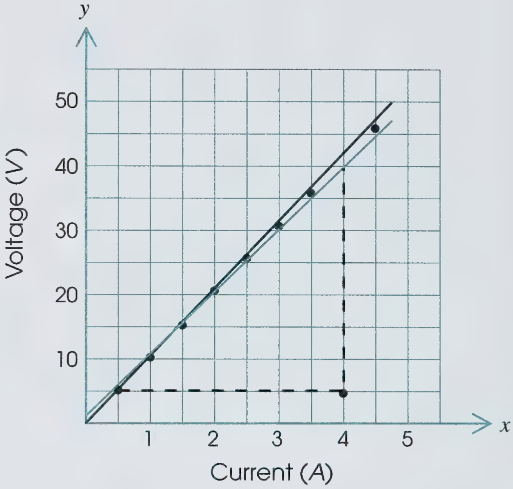
3. Answers may vary.

The y-intercept of the estimated line of best fit is about 1.

Choose two points on the line of best fit to find the slope.

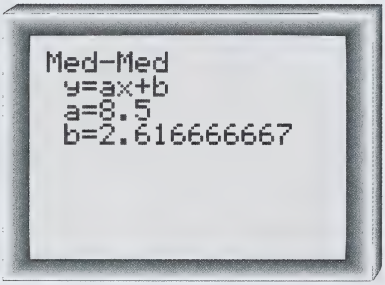
$$\begin{aligned}\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1}\\ &\doteq \frac{40-5}{4-0.4}\\ &\doteq \frac{35}{3.6}\\ &\doteq 9.7\end{aligned}$$

The slope is about 9.7.

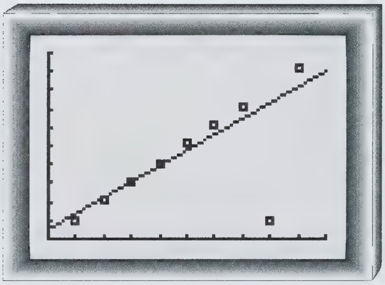


Therefore, the equation of the estimated line of best fit is about $y = 9.7x + 1$.

4. Find the line of best fit using the median-median method. The equation of the line of best fit and the resulting graph are as follows. (Your scatterplot may vary slightly, depending on the WINDOW settings.)

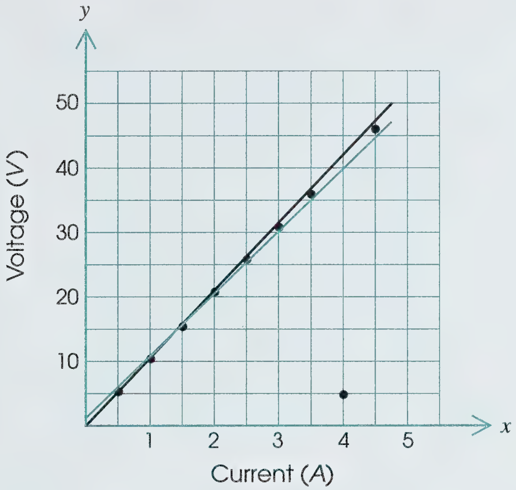


```
Med-Med
 y=ax+b
 a=8.5
 b=2.616666667
```

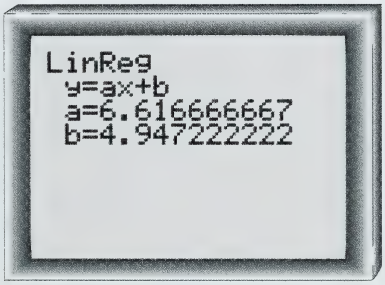


The equation of the median-median line of best fit is about $y = 8.5x + 2.6$.

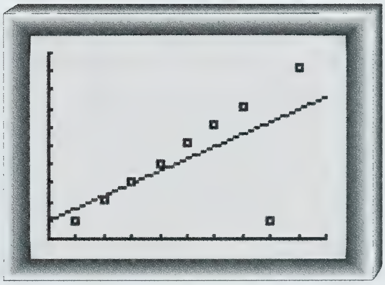
The median-median line of best fit (shown in colour) on the coordinate grid looks as follows. Compare this line to the estimated line (shown in black).



5. Find the line of best fit using the least squares method. The equation of the line of best fit and the resulting graph are as follows.

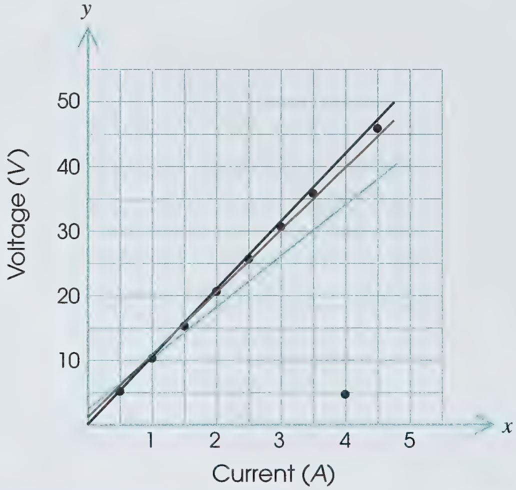


```
LinReg
 y=ax+b
 a=6.616666667
 b=4.947222222
```



The equation of the line of best fit using the least squares method is about $y = 6.6a + 4.9$.

The least squares line of best fit (shown in colour) on the coordinate grid looks as follows. The estimated line of best fit is shown in black; the median-median line of best fit is shown in grey. Compare the three lines.



19.

	Ordered Data	Mean	Median
a.	25 26 27 27 27 30	27	27
b.	25 26 27 27 30 51	31	27
c.	25 26 27 27 30 63	33	27
d.	25 26 27 27 30 99	39	27

20. Textbook exercise 6 of "Investigation 2: Compare the Lines of Best Fit Obtained Using Different Methods," p. 302

6. The outlier $(4.0, 4.7)$ is likely due to an error in measurement rather than an atypical measure.

The line of best fit based on the least squares method is most affected by the outlier, and it is, therefore, least accurate. The estimated line of best fit ignores the outlier to give the most valid picture of the trend. The median-median line is very close to the estimated line.

The estimated line of best fit is closest to going through $(0, 0)$. The line of best fit should go through $(0, 0)$ since the voltage should be zero for a current of zero amperes.

Note: Your answer may vary. What is important here is that you see that one kind of line of best fit may be more appropriate than another for a particular circumstance.

21. Textbook exercises 3, 4, and 6 of "Discussing the Ideas," p. 303

3. Examining the scatterplot may quickly identify outliers. Identifying outliers will help you decide whether the outlier should or should not influence the placement of the line of best fit.

4. Estimating the appropriate domain and range will allow the following:

- All data points will appear on the screen.
- Data points will not be crowded and indistinguishable.

6. You can use the TRACE feature of the calculator to display the coordinates of points on a line. The TRACE feature can also be used to display the coordinates of points of a scatterplot.

22. Textbook exercise "Communicating the Ideas," p. 303

Answers will vary. A sample response is given here.

The line of best fit can be obtained in three ways—using estimation, using the median-median method, and using the method of least squares.

- Estimation involves placing the line of best fit by hand. You place the line so that about half the data points are above the line and half are below the line.

- In the median-median method, a preliminary line is placed through the two outside summary points. The preliminary line is then shifted one-third of the way towards the middle summary point. The parallel, shifted line is the line of best fit according to this method. A graphing calculator may be used to automatically perform the calculations.

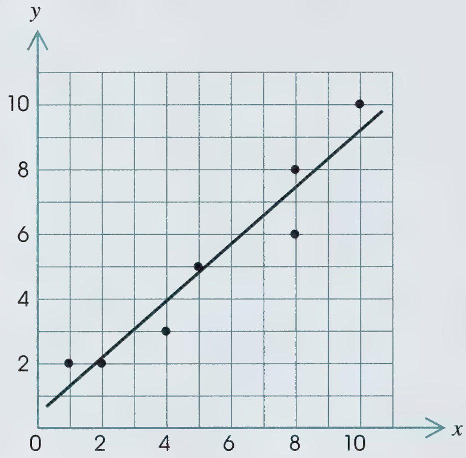
- In the method of least squares, the line of best fit is placed so that the sum of the squares of the residuals of the points is minimized. A graphing calculator is used to automatically perform the calculations required to determine the equation of the line.

Activity 3: The Correlation Coefficient

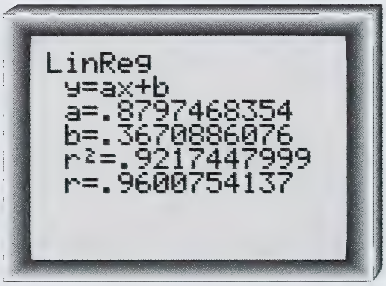
1. **Textbook exercises 1.a., d., f., g., i., and j., and 2 and 3 of "Investigation: The Correlation Coefficient," pp. 305 and 306**

1. to 3. **a.** The scatterplot and estimated line of best fit will look as follows.

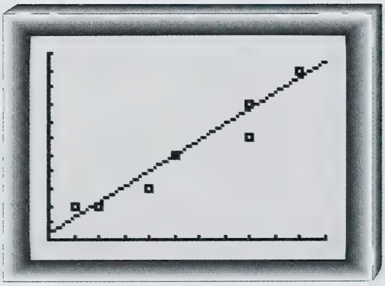
x	1	2	4	5	8	8	10
y	2	2	3	5	6	8	10



The points fit somewhat closely to the line of best fit.



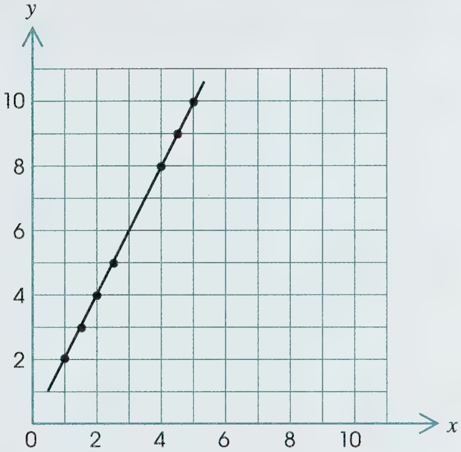
```
LinReg
 y=ax+b
 a=.8797468354
 b=.3670886076
 r²=.9217447999
 r=.9600754137
```



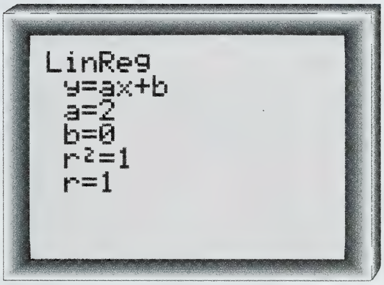
The correlation coefficient, r, is about 0.96. The calculator screen shows the line of best fit using the method of least squares.

d. The scatterplot and estimated line of best fit will look as follows.

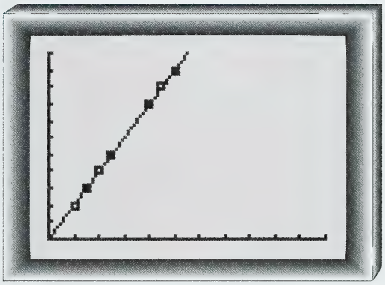
x	1	1.5	2	2.5	4	4.5	5
y	2	3	4	5	8	9	10



The points fit exactly on the line.



```
LinReg
 y=ax+b
 a=2
 b=0
 r²=1
 r=1
```

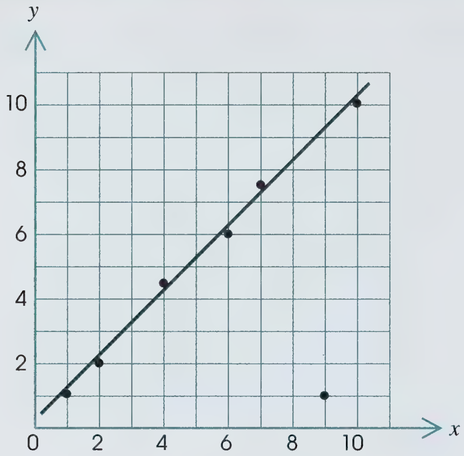


The correlation coefficient, *r*, is 1. The calculator screen shows the line of best fit using the method of least squares.

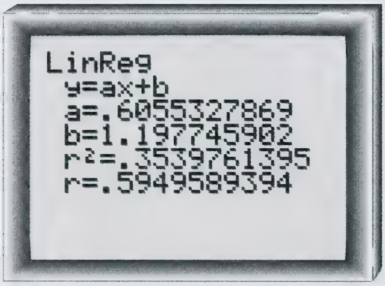
Activity 3 (continued)

f. The scatterplot and estimated line of best fit will look as follows.

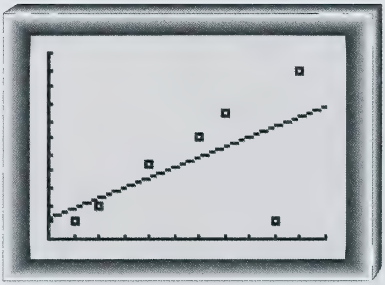
x	1	2	4	6	7	9	10
y	1	2	4.5	6	7.5	1	10



The points fit very closely except for one outlier, $(9, 1)$.



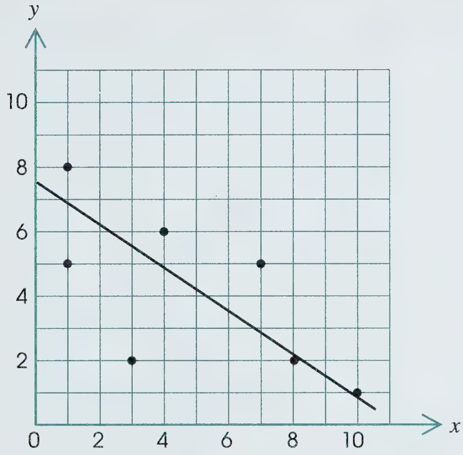
```
LinReg
 y=ax+b
 a=.6055327869
 b=1.197745902
 r²=.3539761395
 r=.5949589394
```



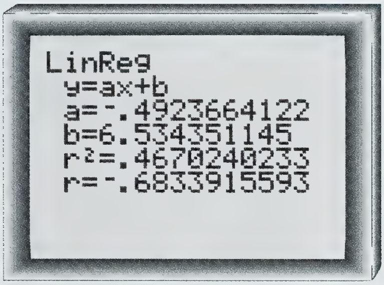
The correlation coefficient, r, is about 0.59. The calculator screen shows the line of best fit using the method of least squares. The calculator takes the outlier into account when calculating values for a, b, r^2, and r.

g. The scatterplot and estimated line of best fit will look as follows.

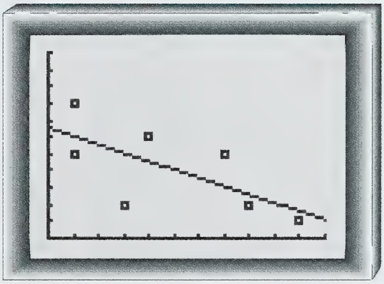
x	1	1	3	4	7	8	10
y	5	8	2	6	5	2	1



The points fit the line weakly.



```
LinReg
 y=ax+b
 a=⁻.4923664122
 b=6.534351145
 r²=.4670240233
 r=⁻.6833915593
```

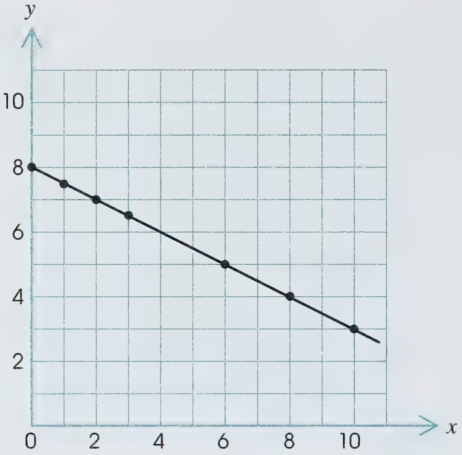


The correlation coefficient, r, is about -0.68. The calculator screen shows the line of best fit using the method of least squares.

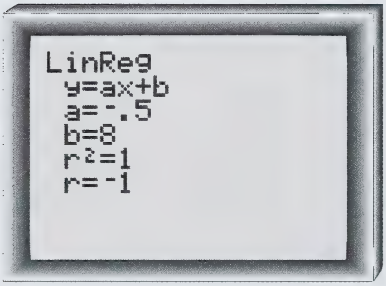
Activity 3 (continued)

i. The scatterplot and estimated line of best fit will look as follows.

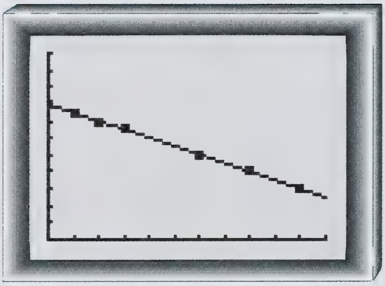
x	0	1	2	3	6	8	10
y	8	7.5	7	6.5	5	4	3



The points fit the line exactly.



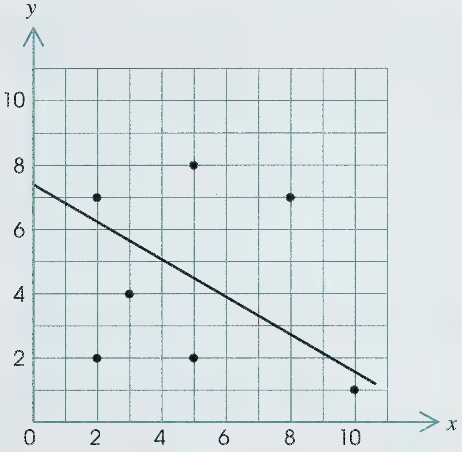
```
LinReg
 y=ax+b
 a=-.5
 b=8
 r²=1
 r=-1
```



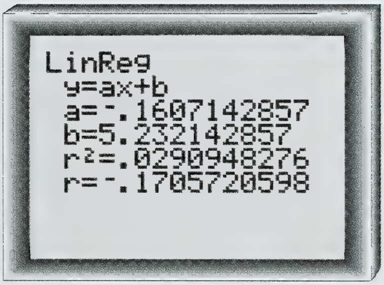
The correlation coefficient, r, is -1. The calculator screen shows the line of best fit using the method of least squares.

j. The scatterplot and estimated line of best fit will look as follows.

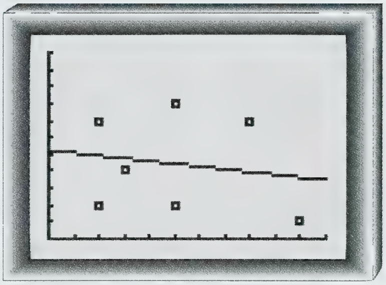
x	2	2	3	5	5	8	10
y	2	7	4	2	8	7	1



The points do not fit the line at all.



```
LinReg
 y=ax+b
 a=⁻.1607142857
 b=5.232142857
 r²=.0290948276
 r=⁻.1705720598
```



The correlation coefficient, r, is about -0.17. The calculator screen shows the line of best fit using the method of least squares.

2. Textbook exercises 5 and 6 of "Investigation: The Correlation Coefficient," p. 306

5. The closer the data points are to the line of best fit, the closer the absolute value of r is to 1 or -1. The farther away the points are from a line of best fit, the closer the value of r is to 0.

6. Graphs with positive slopes are 1.a., 1.d., and 1.f. Graphs 1.g., 1.i., and 1.j. have a negative slope. The sign of the slope of the line of best fit is the same as the sign of the correlation coefficient.

Activity 3 (continued)

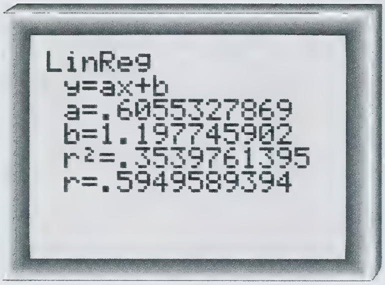
3. Textbook exercises 1 to 5 of "Discussing the Ideas," p. 306

1. The correlation coefficient, *r*, lies between -1 and 1 inclusive. You can express possible values for *r* as $-1 \leq r \leq 1$.

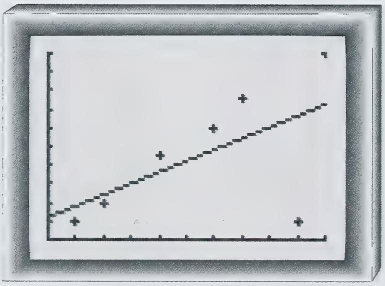
2. If *r* equals 1 or -1, then all the data points are collinear—they fall on the line. If *r* is close to 1 or -1, then the points are very close to a line. If *r* is close to 0, then the points are scattered and are not collectively close to any line.

3. With the outlier moved close to alignment with the other points of the set, the value of *r* nears 1. In the initial graph, *r* is about 0.6; in the modified graph, *r* is almost 1.

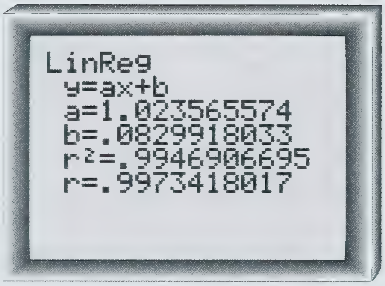
Initial Graph



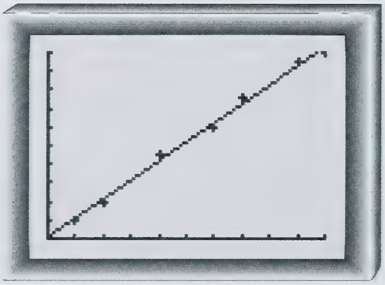
```
LinReg
 y=ax+b
 a=.6055327869
 b=1.197745902
 r²=.3539761395
 r=.5949589394
```



Modified Graph



```
LinReg
 y=ax+b
 a=1.023565574
 b=.0829918033
 r²=.9946906695
 r=.9973418017
```



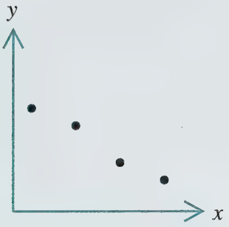
4. The line of best fit of a set of data with only two points is the line through the points. Therefore, two points will always be exactly on the line of best fit. If the slope of the line connecting the points is negative, $r = -1$. If the slope of the line is positive, $r = 1$. If the slope is 0 (that is, the line is horizontal), then the value of *r* is undefined. If the two data points are on a vertical line, then the value of *r* is undefined.

   Note: An *r*-value of -1 does not imply the slope is -1. Also, an *r*-value of $+1$ does not imply the slope is $+1$.

5. If you depend only on the correlation coefficient to judge how close a data set is to a linear function, you may overlook that a poor correlation could be due to just one outlier. The outlier may be due to an error in measurement or some flaw in experimental procedure or some atypical value.

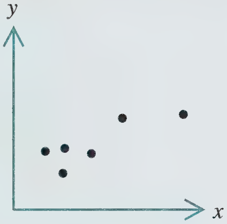
4. Textbook exercises 1.a., 1.d., 2, 4, and 5 of "Exercises: Checking Your Skills," pp. 306 to 308

1. a.



The points will be very close, but not exactly on a line. The points will slope downwards.

d.

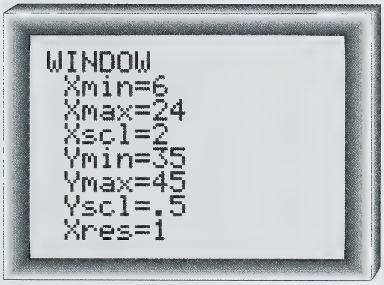


The points will be somewhat scattered, but a linear trend should be recognizable. The points will slope upwards.

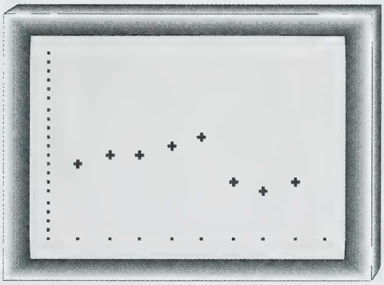
2. Your estimates may vary. The signs should agree.

a. $r \doteq 0$ **b.** $r \doteq -0.95$ **c.** $r \doteq +0.90$ **d.** $r \doteq +0.99$ **e.** $r \doteq -0.95$

4. a. The graph will look like the following. Graphs will vary depending on the chosen WINDOW settings.

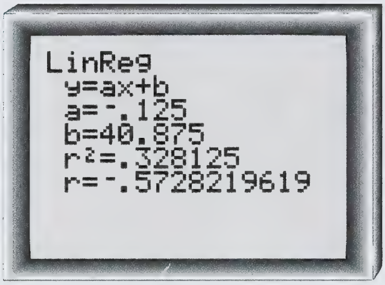


```
WINDOW
 Xmin=6
 Xmax=24
 Xscl=2
 Ymin=35
 Ymax=45
 Yscl=.5
 Xres=1
```

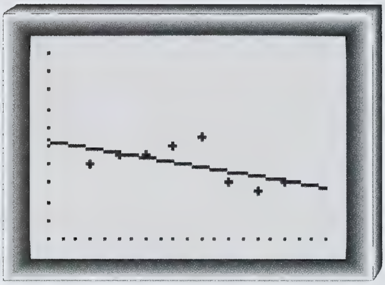


Estimates of r will vary. A reasonable prediction of the value of the correlation coefficient is about -0.6.

b. The value of r is about -0.57. The equation of the line of best fit (based on the method of least squares) is $y = -0.125x + 40.875$.



```
LinReg
 y=ax+b
 a=⁻.125
 b=40.875
 r²=.328125
 r=⁻.5728219619
```



c. The correlation coefficient is about -0.6, which means there is little reason to consider the trend to be linear. There is about a 60% chance that a temperature will be the predicted temperature. Therefore, any prediction based on the line of best fit should only be accepted with caution.

This line of best fit is based on limited data. If the previous few days been included, the line of best fit would appear different—probably much closer to being level.

5. In scatterplot (a), the points are dispersed with only a hint of linearity. The relation between shoe size and marks is weak. In graph (b), the points are closer to forming a line than in graph (a). The relation between study time and marks is quite strong. The r value of scatterplot (b) is closer to 1.

5. The correlation coefficient, r, is number associated with a set of data points that indicates how close the points lie from their line of best fit. The value of r is always between -1 and $+1$ inclusive. The closer $|r|$ is to 1, the better the data is represented by the line of best fit. If $r < 0$ (that is, r is negative), the slope of the line of best fit is negative. If $r > 0$, then the slope of the line of best fit is positive.

Where r is close to 0, the data points are spread in such a way that no linear trend can be identified. This indicates a likelihood that the x- and y-variables are not closely associated or that one or two outliers might be affecting the correlation.

Follow-up Activities

1. Textbook exercises 7.b., 8.b., 10, and 12 of Part B of "What Should I Be Able To Do?" pp. 314 to 316

7. b. Find the x-intercept; $y=0$ at the x-intercept.

$$\begin{aligned} 3x+4y+24&=0 \\ 3x+4(0)+24&=0 && \longleftarrow \text{Substitute the value 0 for } y. \\ 3x&=-24 && \longleftarrow \text{Simplify and subtract 24 from each side.} \\ x&=-8 && \longleftarrow \text{Divide each side by 3.} \end{aligned}$$

The x-intercept is -8.

Find the y-intercept; $x=0$ at the y-intercept.

$$\begin{aligned} 3x+4y+24&=0 \\ 3(0)+4y+24&=0 && \longleftarrow \text{Substitute the value 0 for } x. \\ 4y&=-24 && \longleftarrow \text{Simplify and subtract 24 from each side.} \\ y&=-6 && \longleftarrow \text{Divide each side by 4.} \end{aligned}$$

The y-intercept is -6.

8. b. Find the slope. The slope is the coefficient of x when the equation is in slope y-intercept form.

$$\begin{aligned} 4x-3y-6&=-6 \\ 4x-3y&=0 && \longleftarrow \text{Add 6 to both sides.} \\ -3y&=-4x && \longleftarrow \text{Subtract } 4x \text{ from both sides.} \\ y&=\frac{4}{3}x && \longleftarrow \text{Divide each side by } -3. \end{aligned}$$

The slope y-intercept form of the relation is $y=\frac{4}{3}x+0$.

The slope is $\frac{4}{3}$.

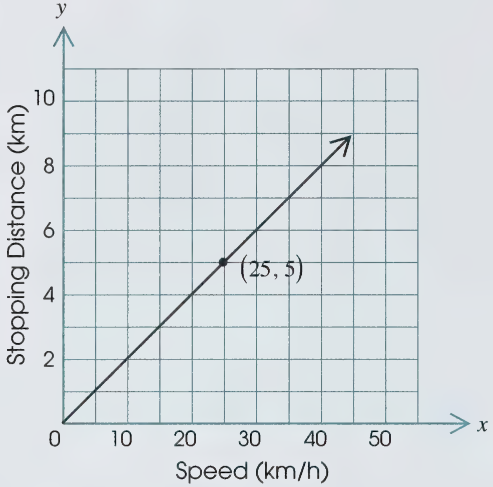
Find the y-intercept; $x=0$ at the y-intercept.

$$\begin{aligned} 4x-3y-6&=-6 \\ 4(0)-3y-6&=-6 \\ -3y&=0 && \longleftarrow \text{Add 6 to both sides.} \\ y&=0 && \longleftarrow \text{Divide each side by } -3. \end{aligned}$$

The y-intercept is 0.

Follow-up Activities (continued)

10. a. If the stopping distance varies directly as speed, the graph is a straight line passing through the origin.



Complete the graph by drawing a line from $(0, 0)$ through $(25, 5)$.

b. Let *d* be the stopping distance in kilometres and *v* be the initial speed in kilometres per hour.

Then, $d = mv$, since $y = mx$ is the general form of a direct variation.

$$5 = m25$$ ← Substitute the coordinates of (25, 5) for the variables *d* and *v*.

$$m = \frac{5}{25}$$ ← Solve for *m*.

$$= \frac{1}{5}$$ ← Simplify.

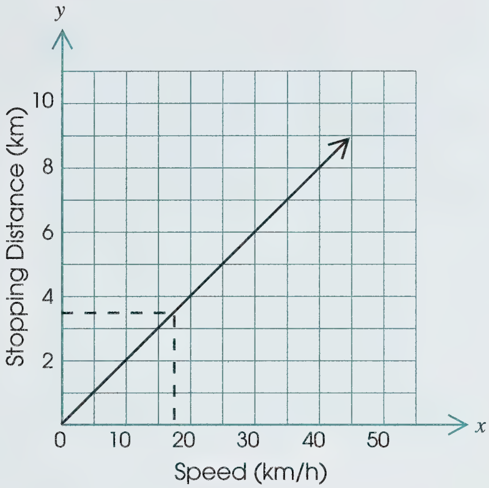
The direct variation is $d = \frac{1}{5}v$.

Find the stopping distance at 12 km/h.

$$\begin{aligned} d &= \frac{1}{5}v \\ &= \frac{1}{5}(12) \\ &= \frac{12}{5} \\ &= 2.4 \end{aligned}$$

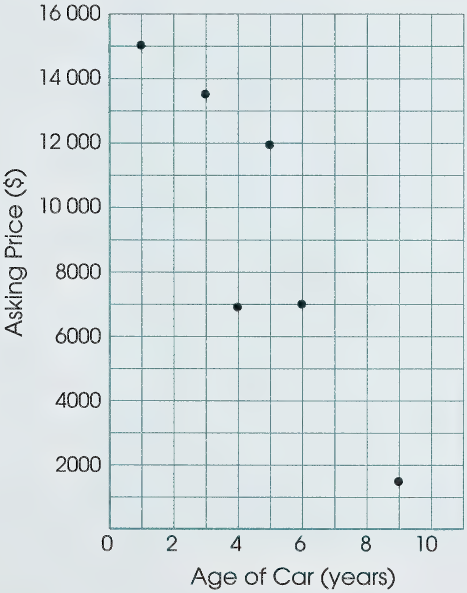
The stopping distance is 2.4 km when the supertanker is travelling at 12 km/h.

c. Find the speed at the point where the graph is at 3.5 km.



The tanker was travelling at a speed of about 17.5 km/h.

12. a. The scatterplot will look like the following.



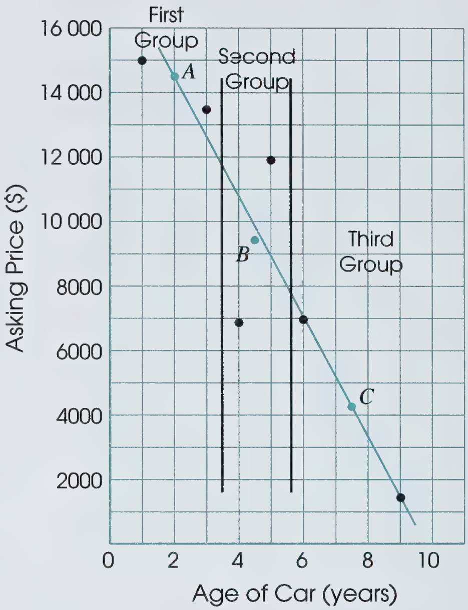
Follow-up Activities (continued)

b. Divide the data points on the graph into three equal groups.

Arrange the data in ascending order for each group.

There is an even number of values in each group.

Determine the median for the x-values and the y-values. This will identify the summary point of each group.



First Group Points: $(1, 15\ 000), (3, 13\ 500)$

Find the median of the x-values.

$$x = \frac{1+3}{2} = \frac{4}{2} = 2$$

Find the median of the y-values.

$$y = \frac{15\ 000 + 13\ 500}{2} = \frac{28\ 500}{2} = 14\ 250$$

The summary point of the first group is $(2, 14\ 250)$.

Second Group Points: $(4, 6900), (5, 11\ 900)$

Find the median of the x-values.

$$x = \frac{4+5}{2} = \frac{9}{2} = 4.5$$

Find the median of the y-values.

$$y = \frac{6900 + 11\ 900}{2} = \frac{18\ 800}{2} = 9400$$

The summary point of the second group is $(4.5, 9400)$.

Third Group Points: $(6, 7000), (9, 1500)$

Find the median of the x-values.

$$\begin{aligned} x &= \frac{6+9}{2} \\ &= \frac{15}{2} \\ &= 7.5 \end{aligned}$$

Find the median of the y-values.

$$\begin{aligned} y &= \frac{7000+1500}{2} \\ &= \frac{8500}{2} \\ &= 4250 \end{aligned}$$

The summary point of the third group is $(7.5, 4250)$.

Place the summary points on the graph and draw a line through the outside points. For this line of best fit, no further calculations are necessary. The line through the outside summary points can be considered to be the best fit line because the middle summary point is almost on this line.

Using the two outside summary points, $(2, 14\ 250)$ and $(7.5, 4250)$, the equation of the line of best fit is determined as follows:

Find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4250 - 14\ 250}{7.5 - 2} \\ &= \frac{-10\ 000}{5.5} \\ &\doteq -1818 \end{aligned}$$

To determine b, use $y = mx + b$ and the coordinates of one of the summary points.

$$\begin{aligned} y &= mx + b \\ 14\ 250 &\doteq -1818(2) + b \\ 14\ 250 &\doteq -3636 + b \\ b &\doteq 17\ 886 \end{aligned}$$

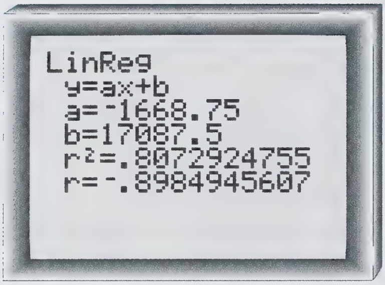
Use the unrounded answer for slope in your calculations.

The equation of the line of best fit is $y = -1818x + 17\ 886$.

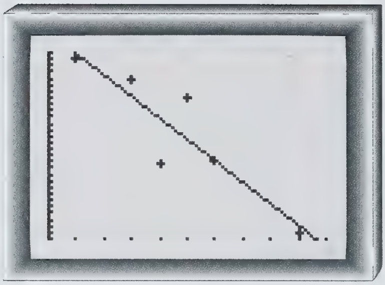
c. Based on a visual inspection, a linear function can describe the data fairly well.

Follow-up Activities (continued)

d.



```
LinReg
 y=ax+b
 a=-1668.75
 b=17087.5
 r²=.8072924755
 r=-.8984945607
```



The correlation coefficient of about -0.9 is very close to -1. This indicates that a linear function can describe the data fairly well. This is consistent with textbook exercise c.

e. According to the line of best fit on the graph, the asking price of a 6-year-old car is approximately \$7000. (Use the TRACE feature on your graphing calculator.) An asking price of \$6000 is less than this.

Whether this price is reasonable depends on several factors: the make and condition of the vehicle, options, mileage, and so on. It is unclear whether values in the problem were gathered on similar vehicles or a cross section of vehicle types. The sample is also very small.

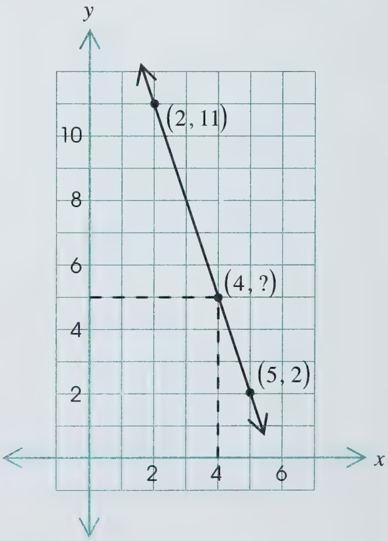
2. Your method of finding the missing value may differ. Following is one method.

On grid paper, graph the points $(2, 11)$ and $(5, 2)$. Then, draw a line through these points.

In order for the coefficient to be -1, the third point must be on the line.

Find the value of the point on the line directly above the $x = 4$ position on the x-axis.

The missing y-value in the chart is 5.



Extra Help

1.

m	4	1	0.5	−0.5
b	0	0	0	0

Note: If you are finding the slope manually, you may use the two points shown on each graph or any points that would lie on the graph to make your calculations.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8-0}{2-0} \\ &= 4 \end{aligned} \qquad \begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8-0}{8-0} \\ &= 1 \end{aligned} \qquad \begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2-0}{4-0} \\ &= 0.5 \end{aligned} \qquad \begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2-0}{4-0} \\ &= -0.5 \end{aligned}$$

a. As the graph is rotated clockwise around the origin, the slope becomes smaller and then becomes negative.

b. The value of m becomes smaller. In the final graph, it is negative.

c. The value of b remains the same, since the y-intercept remains 0.

2.

m	2.0	1.2	0.67	0.20
b	−8.0	−2.0	2.0	8.0

a. As the graph is shifted upwards on the y-axis and rotated clockwise, the slope decreases; in the final position, the line is almost level.

The y-intercept increases; it changes from -8 to $+8$.

b. The value of m decreases to a small positive value.

c. The value of b corresponds to the y-intercept; b increases from -8 to $+8$.

Follow-up Activities (continued)

3.

m	0.25	0.25	0.25	0.25
b	−8.0	−3.0	1.0	6.0

a. As the graph is moved upwards on the y-axis and is kept parallel, the slope remains the same; the y-intercept increases from -8 to $+6$.

b. The value of m does not change.

c. The value of b increases from -8 to $+6$.

4. The slope corresponds to the value of m in $y = mx + b$; the y-intercept corresponds to the value of b.

5. **a.** An outlier above the middle of a set of points raises the line of best fit farther away from the set of points.

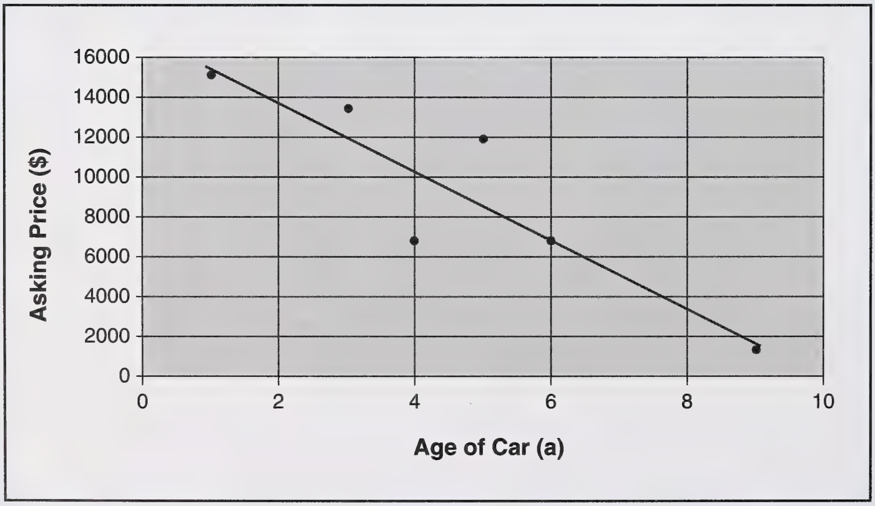
b. An outlier above and to the left causes the left side of the line to tip towards the outlier.

c. An outlier above and to the right causes the right side of the line to tip towards the outlier.

d. An outlier below the middle of the set of points moves the line downward, below the set of points.

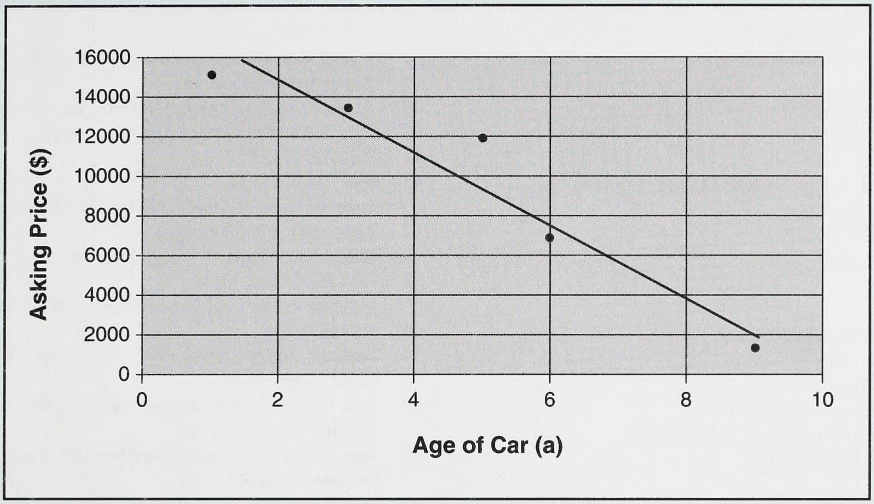
Enrichment

Original Graph



correlation coefficient −0.898 494 561

Modified Graph



correlation coefficient $-0.958\,962\,335$

When you remove the data from the table, the corresponding data point disappears from the scatterplot, the line of best fit moves upwards away from the x-axis, and the correlation coefficient, r, moves closer to -1.

Module Project: Contrasting Two Species

Completing the Project

1. **Textbook exercise 13 of Part C of "What Should I Be Able To Do?" p. 317**

    13. Answers will vary.

    Overall, the graphs plotted by the student are good; however, improvements are possible.

    The student has chosen scales that display well. On graph 1, the data point $(245, 162.5)$ is out of position; the point should be above 245 on the x-axis, rather than above 255. Otherwise, the data is graphed correctly. The line of best fit should be tilted more steeply; there are too many data points above the line in graph 1.

2. Answers may vary. The student may have used a cross to indicate the data point position more precisely; the intersection of the cross arms is a smaller area than the area of a dot.

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